# Circuits Theorem

## 1. Super Position Theorem

The **superposition** principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

- ***Steps to Apply Superposition Principle:***

1. ***Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using the techniques covered in the previous lectures .***
2. ***Repeat step 1 for each of the other independent sources.***
3. ***Find the total contribution by adding algebraically all the contributions due to the independent sources.***

***Example (1): Use the superposition theorem to find v in the circuit of Fig.(6.1).***

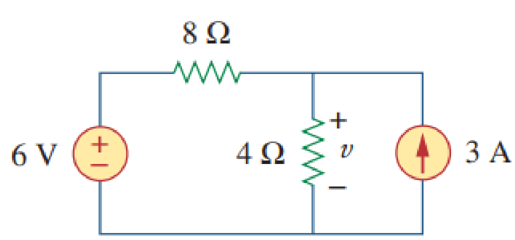



*Figure:(6.1)*

***Solution:***

Since there are two sources, let:

$$\mathbf{v} = \mathbf{v_1} + \mathbf{v_2}$$

where $v_1$ and $v_2$ are the contributions due to the 6-V voltage source and the 3-A current source, respectively. To obtain $v_1$, we set the current source to zero, as shown in Fig. 6.2(a). Applying KVL to the loop in Fig. 6.2(a)

$$12i_1 - 6 = 0 \qquad \Rightarrow \qquad i_1 = 0.5 \text{ A}$$

Thus,

$$v_1 = 4i_1 = 2 \text{ V}$$

We may also use voltage division to get $v_1$ by writing

$$v_1 = \frac{4}{4+8}(6) = 2 \text{ V}$$

$$i_3 = \frac{8}{4+8}(3) = 2 \text{ A}$$

Hence,

$$v_2 = 4i_3 = 8 \text{ V}$$

And we find

$$v = v_1 + v_2 = 2 + 8 = 10 \text{ V}$$

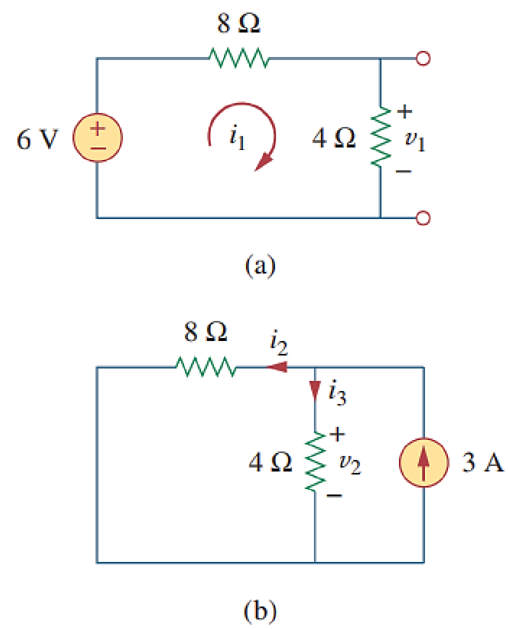



***Figure:(6.2)***

*Example(2): find io in figure (6.3) by superposition theorem:*

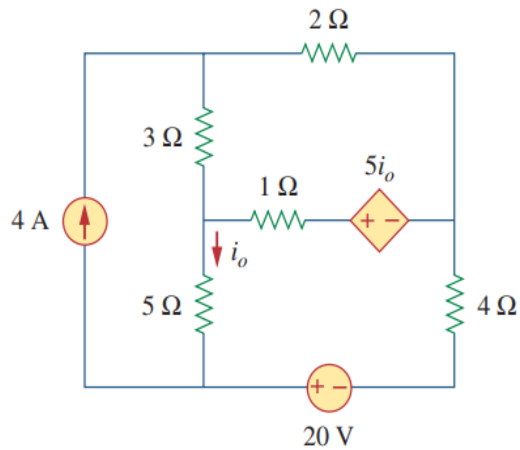



*Figure :(6.3)*

The circuit in Fig. (6.3) involves a dependent source, which must be left intact. We let:

$$i_o = i'_o + i''_o$$

where $i'_o$ and $i''_o$ are due to the 4-A current source and 20-V voltage source respectively. To obtain $i'_o$, we turn off the 20-V source so that we have the circuit in Fig. (a). We apply mesh analysis in order to obtain $i'_o$. For loop 1,

$$i_1 = 4 \text{ A}$$

For loop 2,

$$-3i_1 + 6i_2 - 1i_3 - 5i'_o = 0$$

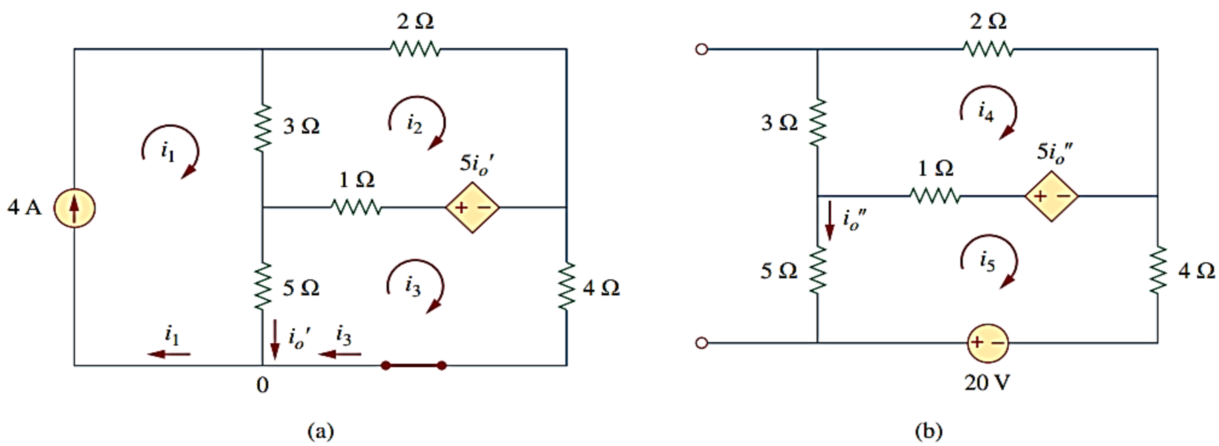



*Figure:(6.4)*

For loop 3,

$$-5i_1 - 1i_2 + 10i_3 + 5i_o' = 0$$

But at node 0,

$$i_3 = i_1 - i_o' = 4 - i_o'$$

$$3i_2 - 2i_o' = 8$$

$$i_2 + 5i_o' = 20$$

which can be solved to get

$$i_o' = \frac{52}{17}\,\text{A}$$

***To obtain $i''_o$, we turn off the 4-A current source so that the circuit becomes that shown in Fig. 6.4(b). For loop 4, KVL gives:***

$$6i_4 - i_5 - 5i''_o = 0$$

and for loop 5,

$$-i_4 + 10i_5 - 20 + 5i''_o = 0$$

But $i_5 = -i''_o$.

$$6i_4 - 4i''_o = 0$$

$$i_4 + 5i''_o = -20$$

which we solve to get

$$i''_o = -\frac{60}{17}\text{ A}$$

$$i_o = -\frac{8}{17} = -0.4706\text{ A}$$

***Example (3): For the circuit in Figure (6.5), use the superposition theorem to find i.***

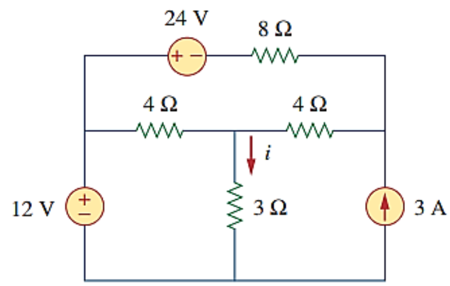



*Figure:(6.5)*

***Solution :***

In this case, we have three sources. Let

$$i = i_1 + i_2 + i_3$$

where $i_1$, $i_2$, and $i_3$ are due to the 12-V, 24-V, and 3-A sources

To get $i_1$, consider the circuit in Fig. 6.6 (a). Combining 4 Ω (on the right-hand side) in series with 8 Ω gives 12 Ω. The 12 Ω in parallel with 4 Ω gives 12 × 4/16 = 3 Ω. Thus,

$$i_1 = \frac{12}{6} = 2 \text{ A}$$

To get $i_2$, consider the circuit in Figure (6.6)(b). Applying mesh analysis gives:

$$16i_a - 4i_b + 24 = 0 \quad \Rightarrow \quad 4i_a - i_b = -6$$

$$7i_b - 4i_a = 0 \quad \Rightarrow \quad i_a = \frac{7}{4}i_b$$

$$i_2 = i_b = -1$$

To get $i_3$, consider the circuit in Fig. 6.6 (c). Using nodal analysis gives:

$$3 = \frac{v_2}{8} + \frac{v_2 - v_1}{4} \quad \Rightarrow \quad 24 = 3v_2 - 2v_1$$

$$\frac{v_2 - v_1}{4} = \frac{v_1}{4} + \frac{v_1}{3} \quad \Rightarrow \quad v_2 = \frac{10}{3}v_1$$

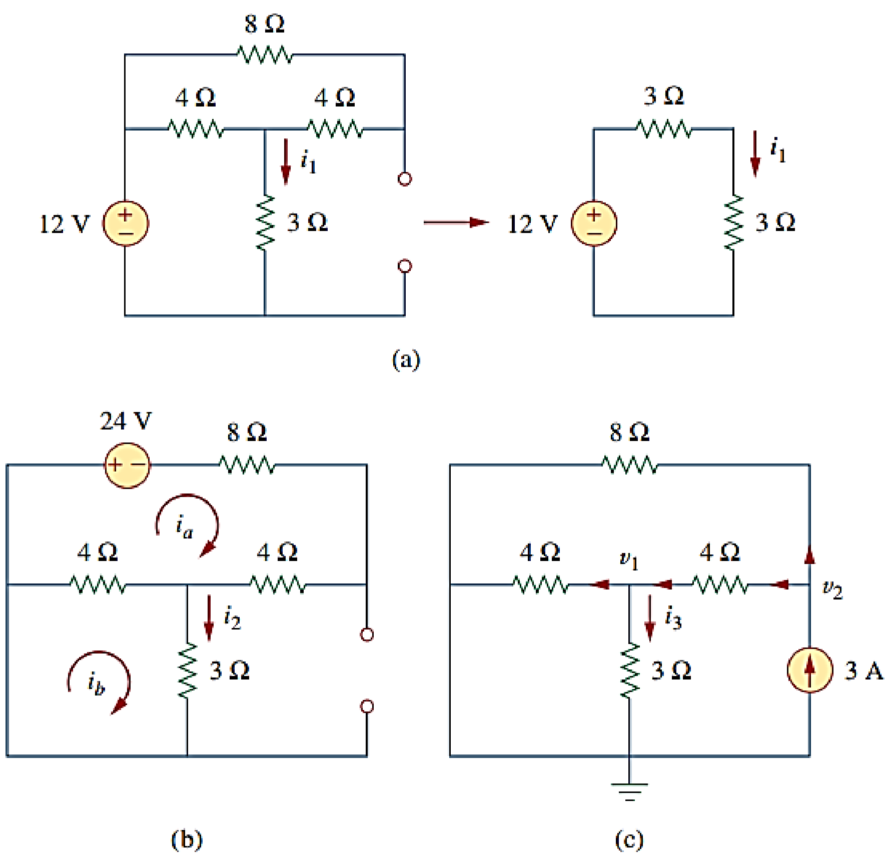



*Figure:(6.6)*

## 2. Thevenin's theorem

**Thevenin's theorem** states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source $V_{Th}$ in series with a resistor $R_{Th}$, where $V_{Th}$ is the open-circuit voltage at the terminals and $R_{Th}$ is the input or equivalent resistance at the terminals when the independent sources are turned off.

According to Thevenin's theorem, the linear circuit in Fig. 6.7(a) can be replaced by that in Fig. 6.7(b). (The load in Fig. 6.7 may be a single resistor or another circuit.) The circuit to the left of the terminals *a*-*b* in Fig. 6.7(b) is known as the ***Thevenin equivalent circuit***.

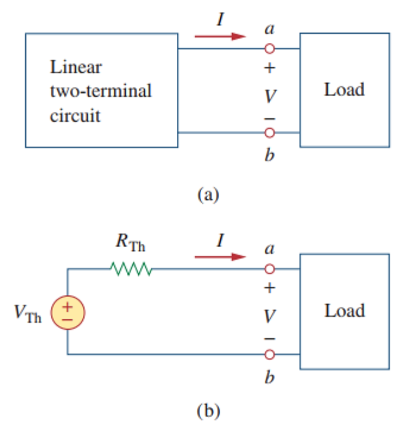



Figure:(6.7)

### ➢ Steps to find $V_{Th}$ and $R_{Th}$

1. ***Remove the portion of the network across which the Thevenin equivalent circuit is to be found .***
2. ***Mark the terminal of the remining tow-terminal network.***
3. ***$R_{Th}$/ Calculate $R_{Th}$by first setting all sources to zero (voltage source are replaced by short circuit and current sources are replaced by open circuits) and finding the resultant resistance between the two marked terminal.***
4. ***$V_{Th}$/ Calculate $V_{Th}$ by first returning all sources to their original positions and finding the open circuit voltage between the marked terminal.***

5. ***Draw the Thevenin's equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.***

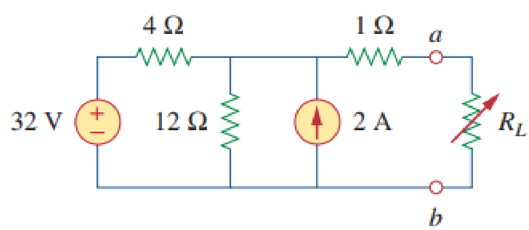
**To find the Thevenin resistance $R_{Th}$, we need to consider two cases:**

**Case 1 :** If the network has no dependent sources, we turn off all independent sources. $R_{Th}$ is the input resistance of the network looking between terminals *a* and *b*, as shown in Fig. 6.7(b).

**Case 2 :** If the network has dependent sources, we turn off all independent sources. As with superposition, dependent sources are not to be turned off because they are controlled by circuit variables.

***Example (4): Find the Thevenin equivalent circuit of the circuit shown in Fig. 6.8, to the left of the terminals a-b. Then find the current through $R_L$ = 6, 16, and 36 Ω.***

*Figure:(6.8)*




# solution:

We find $R_{Th}$ by turning off the 32-V voltage source (replacing it with a short circuit) and the 2-A current source (replacing it with an open circuit). The circuit becomes what is shown in Fig.6.9 ( a). Thus,

$$R_{Th} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\,\Omega$$

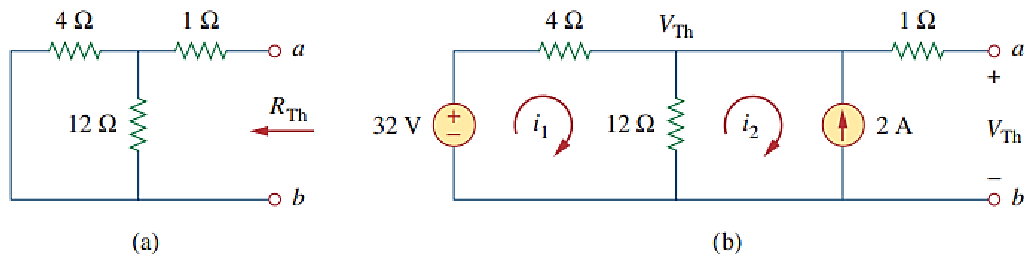



*Figure (6.9) : (a) finding RTh, (b) finding VTh*

To find $V_{Th}$, consider the circuit in Fig. 6.9 (b). Applying mesh analysis to the two loops, we obtain:

$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \qquad i_2 = -2\text{ A}$$

Solving for $i_1$, we get $i_1 = 0.5$ A. Thus,

$$V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30\text{ V}$$

Alternatively, it is even easier to use nodal analysis. We ignore the 1-Ω resistor since no current flows through it. At the top node, KCL gives

$$\frac{32 - V_{Th}}{4} + 2 = \frac{V_{Th}}{12}$$

or

$$96 - 3V_{Th} + 24 = V_{Th} \quad \Rightarrow \quad V_{Th} = 30\text{ V}$$

as obtained before. We could also use source transformation to find $V_{Th.}$ The Thevenin equivalent circuit is shown in Fig. 6.10. The current through $R_L$ is:

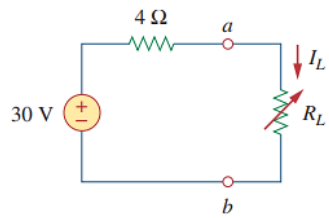



*Figure (6.10): The Thevenin equivalent circuit for Example 4.*

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L}$$

When $R_L = 6$,

$$I_L = \frac{30}{10} = 3 \text{ A}$$

When $R_L = 16$,

$$I_L = \frac{30}{20} = 1.5 \text{ A}$$

When $R_L = 36$,

$$I_L = \frac{30}{40} = 0.75 \text{ A}$$

***Example (5):*** **Find the Thevenin equivalent of the circuit in Fig. 6.11 at terminals *a-b*.**

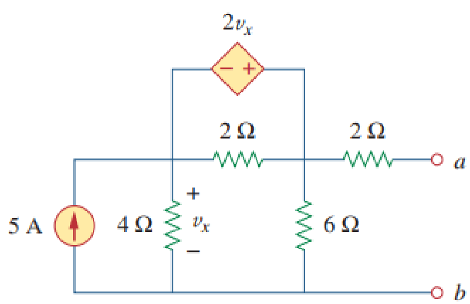



*Figure:(6.11)*

## *Solution :*

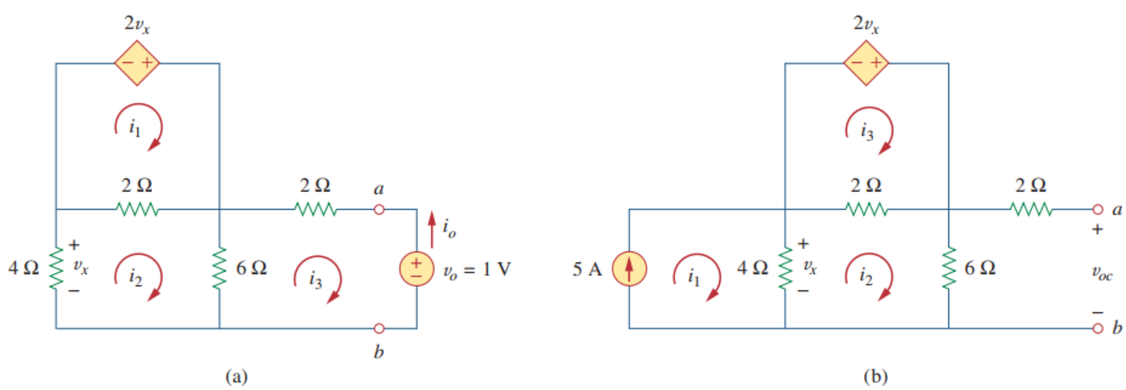



**Figure (6.12): Rth and V th**

Applying mesh analysis to loop 1 in the circuit of Fig. 6.12(a)results in

$$-2v_x + 2(i_1 - i_2) = 0 \qquad \text{or} \qquad v_x = i_1 - i_2$$

But $-4i_2 = v_x = i_1 - i_2$; hence,

$$i_1 = -3i_2$$

For loops 2 and 3, applying KVL produces

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$$

$$6(i_3 - i_2) + 2i_3 + 1 = 0$$

Solving these equations gives

$$i_3 = -\frac{1}{6}\text{ A}$$

But $i_o = -i_3 = 1/6$ A. Hence,

$$R_{Th} = \frac{1\text{ V}}{i_o} = 6\ \Omega$$

To get $V_{Th}$, we find $v_{oc}$ in the circuit of Fig. 6.12(b). Applyingmesh analysis, we get:

$$i_1 = 5$$

$$-2v_x + 2(i_3 - i_2) = 0 \qquad \Rightarrow \qquad v_x = i_3 - i_2$$

$$4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$$

or

$$12i_2 - 4i_1 - 2i_3 = 0$$

But $4(i_1 - i_2) = v_x$. Solving these equations leads to $i_2 = 10/3$. Hence,

$$V_{Th} = v_{oc} = 6i_2 = 20\text{ V}$$

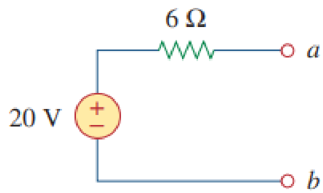
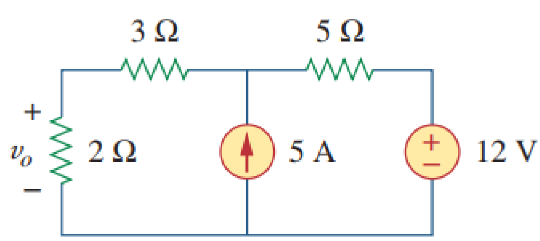



*Figure (6.13): The Thevenin equivalent of the circuit in example 5.*

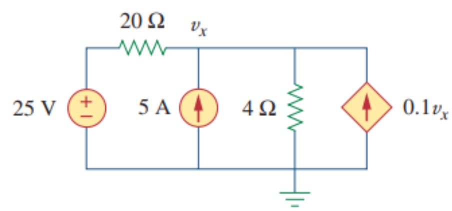
# Problems

***Q1: find Vo in the figure below by super position theorem.***



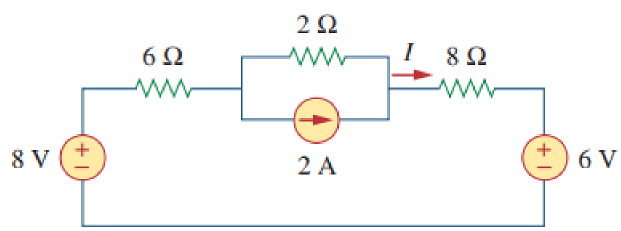

***Vo=7.4v***

***Q2: by superposition theorem find Vx:***



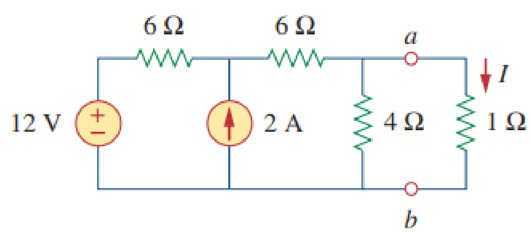

***Vx=31.25v***

***Q3: Find I in the circuit in the figure below using the superposition principle.***




***I=375 mA***

***Q4:*** **Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit Then find *I*.**




**Answer:** $V_{Th} = 6$ V, $R_{Th} = 3\ \Omega$, $I = 1.5$ A.

**Q5: Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit. Then find *I*.**