Norton's Theorem and Maximum Power Transfer

1. Norton's Theorem

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

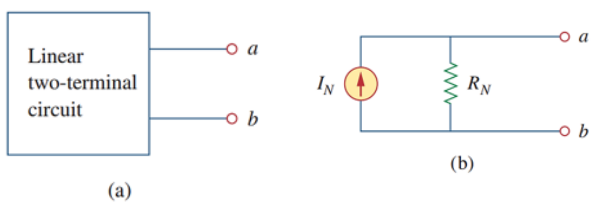


Figure (1) : (a) Original circuit, (b) Norton equivalent circuit.

To find the Norton current I_N, we determine the short-circuit current flowing from terminal a to b in both circuits in Fig.(1). It is evident that the short-circuit current in Fig. (1) (b) is I_N. This must be the same short-circuit current from terminal a to b in Fig. (1)(a), since the two circuits are equivalent. Thus,

$$I_N = i_{sc}$$

Observe the close relationship between Norton's and Thevenin's theorems: $R_N = R_{Th}$, and

$$R_N = R_{Th} \qquad I_N = \frac{V_{Th}}{R_{Th}}$$

To determine the Thevenin or Norton equivalent circuit requires that we find:

- The open-circuit voltage v_{oc} across terminals a *and b.*
- The short-circuit current i_{sc} at terminals *a* and *b.*
- The equivalent or input resistance R_{in} at terminals *a* and when all independent sources are turned off.

$$V_{\text{Th}} = v_{oc}$$

$$I_N = i_{sc}$$

$$R_{\text{Th}} = \frac{v_{oc}}{i_{sc}} = R_N$$

Example (1): Find the Norton equivalent circuit of the circuit in Fig. (2), at terminals a-b.

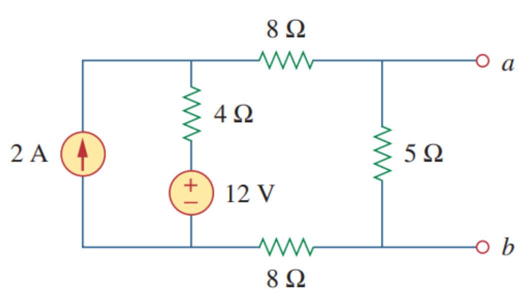


Figure:(2)

Solution:

$$R_N = 5 \parallel (8 + 4 + 8) = 5 \parallel 20 = \frac{20 \times 5}{25} = 4\ \Omega$$

$$i_1 = 2\text{ A}, \qquad 20i_2 - 4i_1 - 12 = 0$$

From these equations, we obtain

$$i_2 = 1\text{ A} = i_{sc} = I_N$$

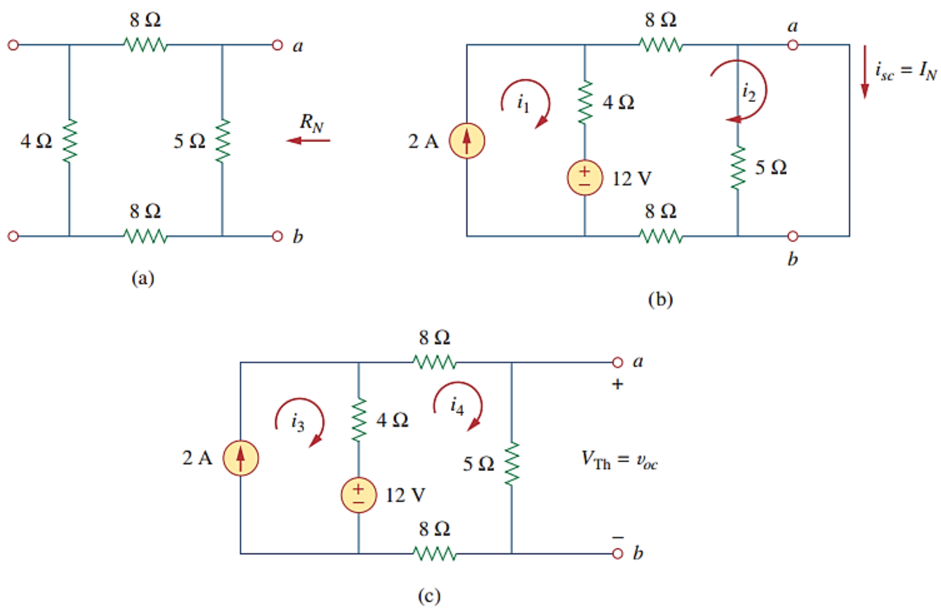


Figure: (3)

$$i_3 = 2 \text{ A}$$

$$25i_4 - 4i_3 - 12 = 0 \quad \Rightarrow \quad i_4 = 0.8 \text{ A}$$

and

$$v_{oc} = V_{Th} = 5i_4 = 4 \text{ V}$$

Hence,

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{4}{4} = 1 \text{ A}$$

Example (2) : Using Norton's theorem, find R_N and I_N of the circuit in Fig. (4) at terminals a-b.

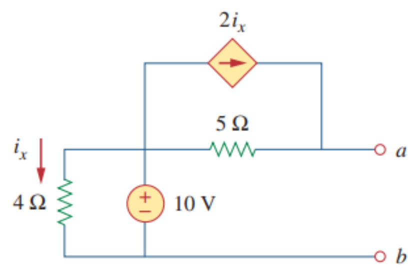


Figure:(4)

$$i_x = \frac{10}{4} = 2.5 \text{ A}$$

At node a, KCL gives

$$i_{sc} = \frac{10}{5} + 2i_x = 2 + 2(2.5) = 7 \text{ A}$$

Thus,

$$I_N = 7 \text{ A}$$

$$R_N = \frac{v_o}{i_o} = \frac{1}{0.2} = 5\ \Omega$$

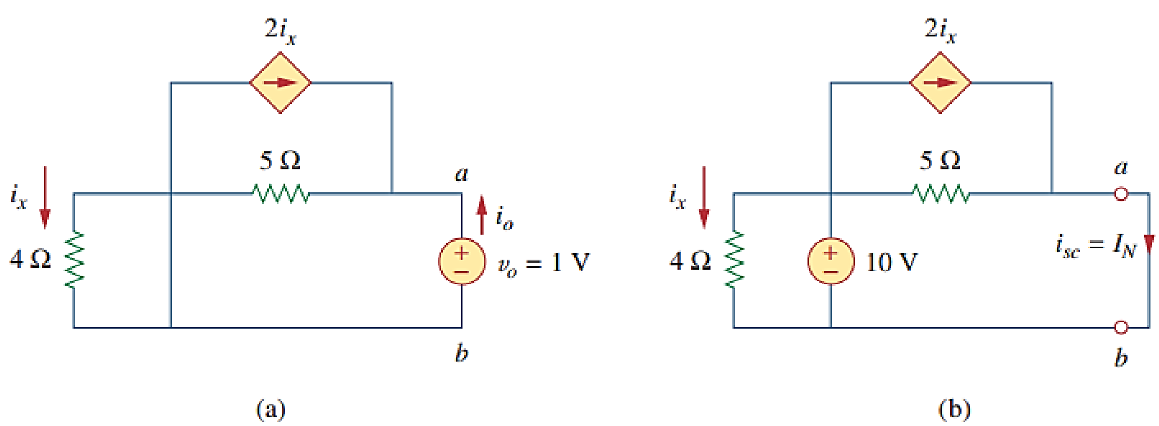


Figure:(5)

2. *Maximum Power Transfer*

Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).

$$R_L = R_{Th}$$

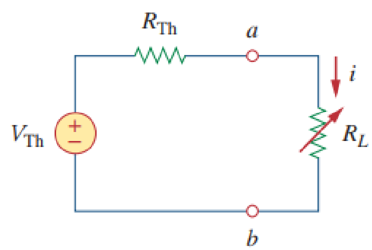


Figure (6): The circuit used for maximum power transfer.

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L \quad \text{………… (1)}$$

$$p_{max} = \frac{V_{Th}^2}{4R_{Th}} \quad \text{……………….(2)}$$

Example (3): Find the value of R_L for maximum power transfer in the circuit of Fig. (7). Find the maximum power.

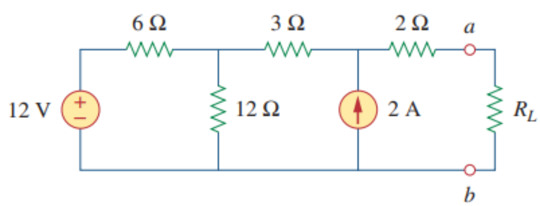
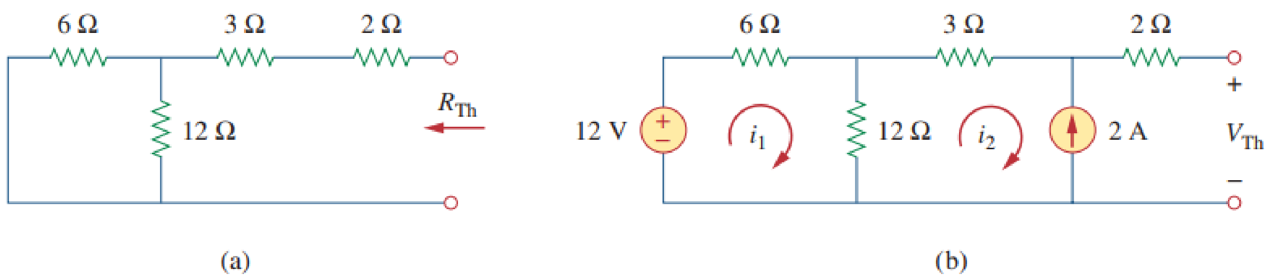


Figure:(7)

Solution :

$$R_{Th} = 2 + 3 + 6 \| 12 = 5 + \frac{6 \times 12}{18} = 9\ \Omega$$



Figure(8): (a) finding** R_{Th}**, (b) finding** V_{Th}**.

$$-12 + 18i_1 - 12i_2 = 0, \qquad i_2 = -2\text{ A}$$

Solving for i_1, we get $i_1 = -2/3$. Applying KVL around the outer loop to get V_{Th} across terminals *a-b*, we obtain

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0 \quad \Rightarrow \quad V_{Th} = 22\text{ V}$$

For maximum power transfer,

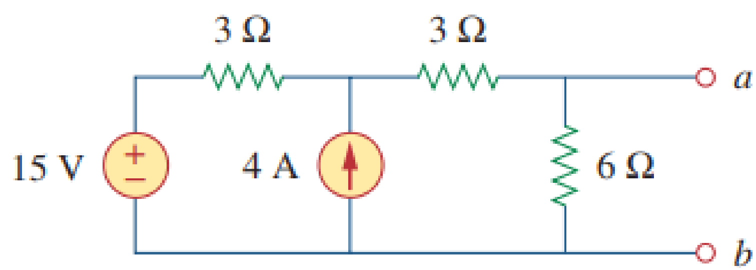
$$R_L = R_{Th} = 9\ \Omega$$

and the maximum power is

$$p_{max} = \frac{V_{Th}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44\text{ W}$$

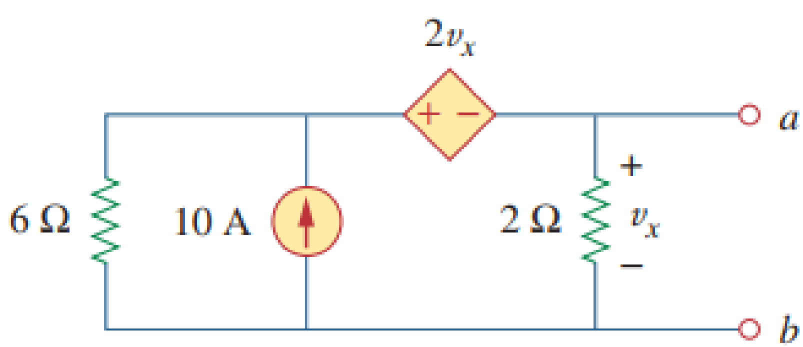
Problems

1. *Find the Norton equivalent circuit for the circuit in Figure below at terminals a-b.*



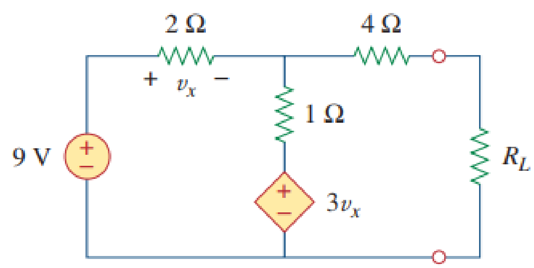
Answer: $R_N = 3\ \Omega, I_N = 4.5$ A.

2. *Find the Norton equivalent circuit of the circuit in Figure below at terminals a-b*



Answer: $R_N = 1\ \Omega, I_N = 10$ A.

3. ***Determine the value of R_L that will draw the maximum power from the rest of the circuit in Figure below. Calculate the maximum power.***



Answer: 4.222 Ω, 2.901 W.