# 2 FRICTION

The surface (even smooth one) of any object is irregular as it has protrusions and valleys.

When two surfaces are in contact, their irregularities intermesh, and as a result, there is a **resistance** to the sliding or moving of one surface on the other.

This resistance is called **friction**. If one surface is to be moved with respect to another, a force must be applied to overcome friction.

The frictional property of surfaces is represented by the **coefficient of friction** (μ):

$$\mu = \frac{F_f}{F_n}$$

$F_f$ is frictional reaction force and $F_n$ is force perpendicular to the surface.

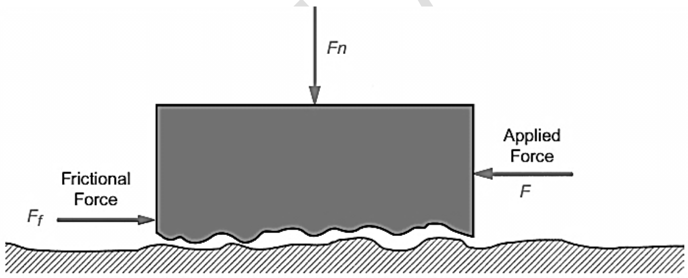


Figure 1: Force components in friction.

The magnitude of the frictional force does **not depend on the size of the contact area.** If the surface contact area is increased, the force per unit area (pressure) is decreased, and this reduces the inter-penetration of the irregularities. At the same time, **the number of irregularities** is proportionately increased.

As a result, the total frictional force is **unchanged**. **Without friction** we could not walk; nor balance on an inclined plane.

Friction is greatly **reduced** by introducing a fluid such as oil at the interface of two surfaces. The fluid fills the irregularities and therefore smooths out the surfaces. **A natural example of such lubrication** occurs in the joints of animals and humans, which are lubricated by a fluid called the **synovial fluid**.

## 2.1 STANDING AT AN INCLINE

Referring to Fig. 1, let us calculate the angle of incline $\theta$ of an oak board (with $\mu_s$= 0.6) on which a person of weight $W$ can **stand without sliding down**.

The force $F_n$ normal to the inclined surface is:

$$F_n = W cos\theta$$

The **static** frictional force $F_f$ is:

$$F_f = \mu F_n = \mu_s W cos\theta = 0.6 W cos\theta$$

The **force parallel to the surface** $F_p$, which tends to cause the sliding, is:

$$F_p = W sin\theta$$

The person will slide when the force $F_p$ is greater than the frictional force $F_f$; that is,

$$F_p > F_f$$

**At the onset of sliding**, these two forces are just equal; therefore,

$$F_f = F_p$$

$$0.6 W cos\theta = W sin\theta$$

or

$$\frac{sin\theta}{cos\theta} = tan\theta = 0.6 \Rightarrow \theta = 31°$$

Table 1: Coefficients of friction, static ($\mu_s$) and Kinetic ($\mu_k$).

| **Surfaces** | $\mu_s$ | $\mu_k$ |
|---|---|---|
| Leather on oak | 0.6 | 0.5 |
| Rubber on dry concrete | 0.9 | 0.7 |
| Steel on ice | 0.02 | 0.01 |
| Dry bone on bone | | 0.3 |
| Bone on joint, lubricated | 0.01 | 0.003 |

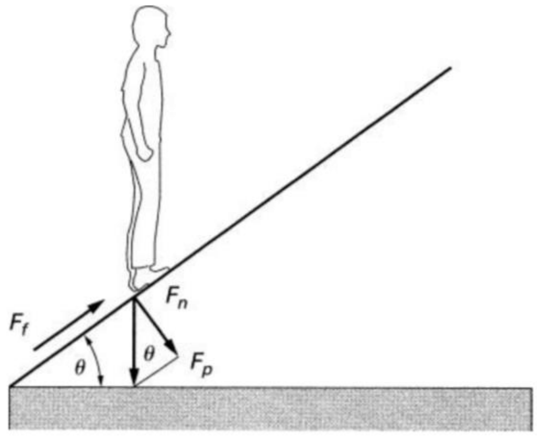


Fig. 1: Standing on an incline.

## 2.2 FRICTION AT THE HIP JOINT

When the joints are in motion, large forces produce frictional wear, which could be **damaging** unless the joints are well lubricated. Frictional wear at the joints is greatly reduced by a smooth **cartilage coating** at the contact ends of the bone and by **synovial fluid** which lubricates the contact areas (see Fig. 2 and Fig. 3).

The frictional force on the joint is:

$$F_f \approx 2.4W\mu$$

The work expended in sliding the joint against this friction during each step:

$$Work = F_f \times distance$$

If the joint were not lubricated, the coefficient of friction would be about 0.3. Under these conditions, the work expended would be:

$$Work = 2.4 \times W \times 0.3 \times 3 \text{ cm} \approx 2.16W\mu$$

This work would be dissipated into heat energy, which would destroy the joint. As it is, the joint is well lubricated, and the coefficient of friction is only 0.003. Therefore, the work expended in counteracting friction and the resultant heating of the joint are negligible.

With ageing, the joint cartilage begins **to wear**, efficiency of lubrication decreases, and the joints may become seriously damaged (see Fig. 3).

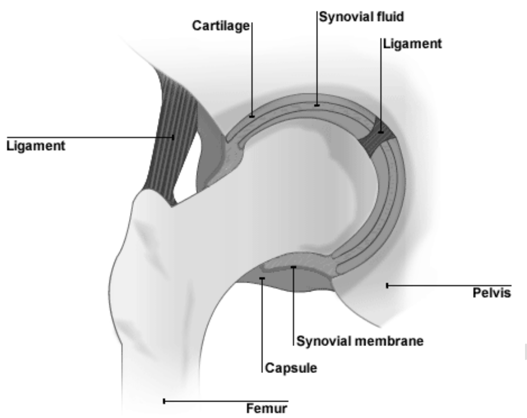


Fig. 2: Lubrication of hip joints by cartilage and synovial fluid.

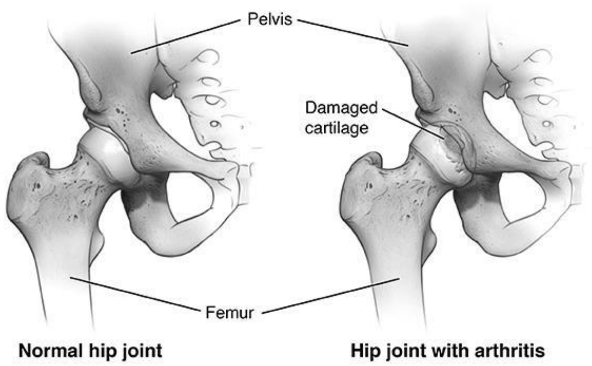


Fig. 3: Normal and damaged hip joints.

## 2.3 VERTICAL JUMP

We will calculate here the height $H$ attained by the jumper. In the crouched position, at the start of the jump, the center of gravity is lowered by a distance $c$.

Thus, there are **two forces acting on the jumper**: his weight ($W$), which is in the downward direction, and the reaction force ($F$) which is in the upward direction. The force F that accelerates the body upward depends on the strength of the leg muscles. The **net upward force** on the jumper is $F - W$.

This force acts on the jumper until his body is erect and his feet leave the ground. The upward force, therefore, acts on the jumper through a distance $c$ (see Fig. 4).

The height of the jump is:

$$H = \frac{(F - W)\,c}{W}$$

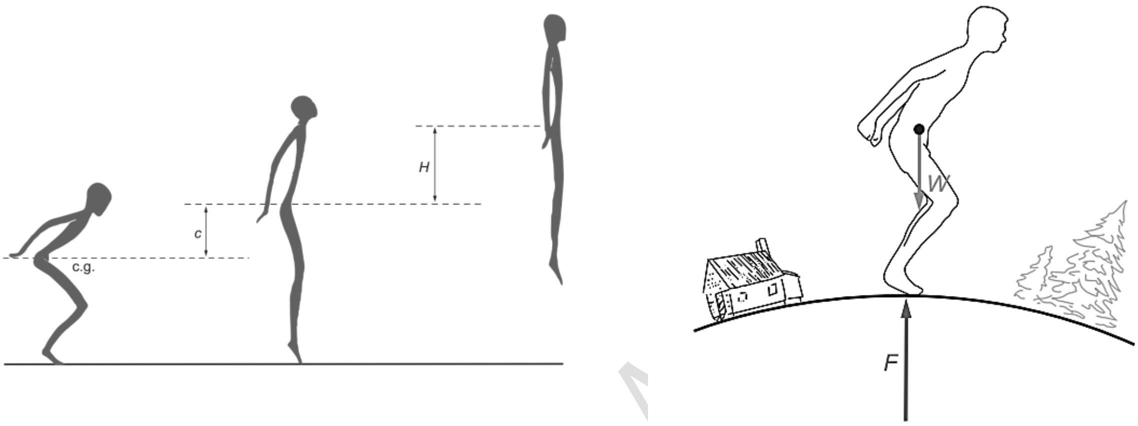


Fig. 4: Vertical jump and forces on the jumper

## 2.4 ELASTICITY AND STRENGTH OF MATERIALS

When a force is applied to a body, the shape and size of the body **change**. Depending on how the force is applied, the body may be stretched, compressed, bent, or twisted. **Elasticity** is the property of a body that tends to return the body to its original shape after the force is removed.

### 2.4.1 Bone Fracture

**Knowledge of the maximum energy that parts of the body can safely absorb** allows us to estimate the possibility of injury under various circumstances. We shall first calculate the amount of energy required to break a bone of area A and length $l$.

Let us designate the breaking stress of the bone as $S_B$ (see Fig. 5). The corresponding force $F_B$ that will fracture the bone is

$$F_B = S_B A = YA\frac{\Delta l}{l}$$

**The energy stored in the compressed bone at the point of fracture** is

$$E = \frac{1}{2}\frac{A\,l\,S_B^2}{Y}$$

This is the amount of energy in the impact of a person with mass $m$ jumping from a height of $h$, given by the product

$$E = mgh$$

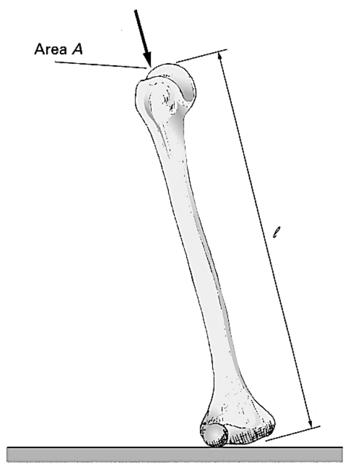


Fig. 5: Compression of a bone.

### 2.4.2 Airbags: Inflating Collision Protection Devices

An inflatable bag is located in the dashboard of the car as shown in Fig. 6. In a collision, the bag expands suddenly and cushions the impact of the passenger. The forward motion of the passenger must be stopped in about 30 cm of motion if contact with the hard surfaces of the car is to be avoided.

The average force that produces the deceleration is

$$F = ma = \frac{mv^2}{2s}$$

where $m$ is the mass of passenger, $a$ is the acceleration of the vehicle, $v$ is the velocity of the vehicle and $s$ is the allowed stopping distance. The necessary stopping force increases as the square of the velocity.

For example, a person with 70 kg weight with allowed stopping distance of 30 cm, the average force of stopping would be

$$F = \frac{mv^2}{2s} = \frac{70\text{ kg} \times (19.5\text{ m/sec})^2}{2 * 0.3\text{ m}} = 4.4 \times 10^4\text{ N}$$

If this force is uniformly distributed over a 0.1 $m^2$ area of the passenger's body, the applied force per $m^2$ is $4.4\times10^6$ N. This is just below the estimated strength of body tissue ($\sim5\times10^5$ N/$m^2$).

Fig. 6: Inflating collision protective device.