

## CHAPTER TWO: FLUID FLOW

### Description of fluid flow

There are two methods to describe the fluid flow

- 1- Eulerian description : this description include observing the fluid velocity at fixed positions (use of velocity fields and flow fields) such as flow through a constant-cross section pipe, as shown in Fig. In the figure, the fluid velocity is shown across the pipe cross section and at various points along the length of the pipe.

A Eulerian description considers a specific region (or control volume) in the flow, and it measures the motion or a fluid property of all the particles that pass through this region.

- 2- Lagrangian description: fluid describes by observing the trajectories of specific fluid parcels اجزاء. This method most efficient way to sample a fluid flow and the physical conservation laws are inherently بشكل متاصل Lagrangian since they apply to moving fluid volumes rather than to the fluid that happens to be present at some fixed point in space.

A Lagrangian description follows the motion of a single particle as the particle, moves through the flow field.

### Types of Fluid Flow

- Classification of Flow Related to Its Frictional Effects.

Laminar Flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream-lines and all the streamlines are straight and parallel. Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called stream-line flow or viscous flow, and occurs at low velocity and/or high viscosity fluid.

Turbulent flow: Turbulent flow is that type of flow in which the fluid particles move in a zig-zag way, causing mixing of the fluid particles and eddies formation, therefore more internal friction than laminar flow. These eddies are responsible for high energy loss. For a pipe flow, the type of flow is determined by a non-dimensional number  $uD/\nu$  called the Reynold number.

Where D= Diameter of pipe

V= Mean velocity of flow in pipe

$\nu$  = Kinematic viscosity of fluid.

If the Reynolds number is less than 2000, the flow is called laminar. If the Reynolds number is more than 4000, it is called turbulent flow. If the Reynolds number lies between 2000 and 4000, the flow may be laminar or turbulent.

Transitional flow: is a state in which coexist متواجدة معا regions of both laminar and turbulent flow. Therefore it exists between them.

### Classification of Flow Based on Space and Time

**Steady and Unsteady Flows** Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density, etc. at a point do not change with time. Thus for steady flow, mathematically, we have

$$\left(\frac{\partial \text{Velocity}}{\partial t}\right)_{x_0, y_0, z_0} = 0, \quad \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0, \quad \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

where  $(x_0, y_0, z_0)$  is a fixed point in fluid field.

Unsteady flow is that type of flow, in which the velocity, pressure density at a point changes with respect to time. Thus, mathematically,

$$\text{for unsteady flow } \left(\frac{\partial \text{Velocity}}{\partial t}\right)_{x_0, y_0, z_0} \neq 0, \quad \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \quad \text{etc.}$$

**Uniform and Non-uniform Flows** Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space (*i.e.*, length of direction of the flow). Mathematically,

$$\text{for uniform flow } \left(\frac{\partial \text{Velocity}}{\partial s}\right)_{t = \text{constant}} = 0$$

where  $\partial s$  = Length of flow in the direction  $s$ .

Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space. Therefore for non-uniform flow

$$\left(\frac{\partial \text{Velocity}}{\partial s}\right)_{t = \text{constant}} \neq 0$$

### Classifications of Dimensional Flow

Three-dimensional flow: if all three space coordinates are required to describe the flow.

Examples include the flow of water around a submarine and the airflow around an automobile.

Two-dimensional flow: if two space coordinates are required to describe the flow. For many problems in engineering, we can simplify the analysis by assuming this type of flow. For example, the flow through the converging pipe in Fig. 3-4.a where the velocity of any particle depends only on its axial and radial coordinates  $x$  and  $r$ .

One-dimensional flow: if one space coordinate is needed to analyze the flow. It is very simple type of flow such as unchanging flow through a uniform straight pipe.

Non-dimensional flow is the unchanging flow of an *ideal fluid*, where the viscosity is zero and the fluid is incompressible, then its velocity profile is *constant* throughout, and therefore independent of its coordinate location.

**Streamlines:** A streamline is defined as the line drawn tangent to the velocity vector at each point in a flow field. By definition, there is no flow across a streamline, *only along* the streamline.

### Classifications of Flow According to Compressibility

**Compressible and Incompressible Flows:** Compressible flow is that type of flow in which the density of fluid changes from point to point or in other words ( $\rho$ ) is not constant for the fluid. Thus, mathematically, for compressible flow

$$\rho \neq \text{constant}$$

Incompressible flow is that type of flow in which the density is constant for the fluid flow. Liquids are generally incompressible while gases are compressible. Thus, mathematically, for incompressible flow

$$\rho = \text{Constant}$$

### Classification of Flow Based on Rotation

**Rotational and Irrotational flow :** Rotational flow is that type of flow in which the fluid particles while flowing along stream-lines, also rotate about their own axis. And if the fluid particles while flowing along stream-lines, do not rotate about their own axis that type of flow is called irrotational flow.

### RATE OF FLOW $m'$ OR DISCHARGE (Q):

It is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel. For an incompressible fluid (or liquid) the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second Q. For compressible fluids, the rate of flow is usually expressed as the mass of fluid flowing across the section  $m'$ . Thus

(i) For liquids the units of Q are  $m^3/s$  or liters/s

(ii) For gases the units of  $m'$  are kg/s

Consider a fluid flowing through a pipe in which

A = Cross-sectional area of pipe. and  $u$  = Average velocity of fluid across the section

Then discharge  $Q = A * u$

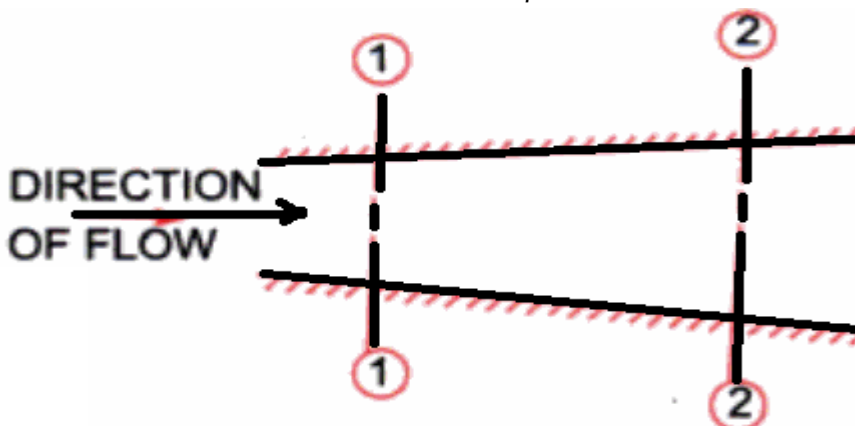
### CONTINUITY EQUATION:

The equation based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant. Consider two cross-sections of a pipe as shown in figure.

Let  $u_1$  = Average velocity at cross-section at 1-1,  $\rho_1$  = Density at section 1-1

$A_1$  = Area of pipe at section 1-1 And  $u_2, \rho_2, A_2$  are corresponding values at section 2-2

Then rate of flow at section 1-1 =  $u_1 \rho_1 A_1$  and Rate of flow at section 2-2 =  $u_2 \rho_2 A_2$



According to law of conservation of mass

Rate of flow at section 1-1 = Rate of flow at section 2-2

$$\rho_1 A_1 u_1 = \rho_2 A_2 u_2 \dots\dots\dots(1)$$

The above equation is applicable to the compressible as well as incompressible fluids is called Continuity Equation. If the fluid is incompressible, then  $\rho_1 = \rho_2$  and continuity equation (1) reduces to

$$A_1 u_1 = A_2 u_2$$

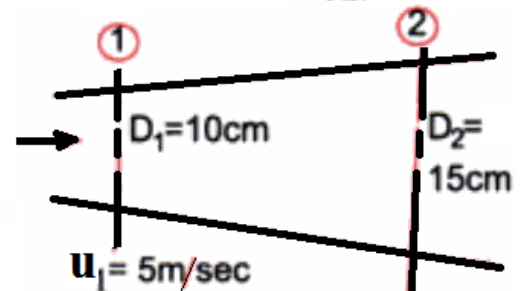
**Example:** The diameters of a pipe at the sections 1 and 2 are 10cm and 15cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5m/s. Determine the velocity at section 2.

**Solution:**  $A_1 = \frac{\pi}{4} (D_1)^2 = \frac{\pi}{4} (.1)^2 = .007854 \text{ m}^2$      $A_2 = \frac{\pi}{4} (.15)^2 = 0.01767 \text{ m}^2$

Discharge through pipe  $Q = A_1 * u_1 = .007854 \times 5 = 0.03927 \text{ m}^3/\text{s}$ .

But  $Q = A_1 u_1 = A_2 u_2$

$$u_2 = \frac{A_1 u_1}{A_2} = \frac{.007854}{.01767} \times 5.0 = 2.22 \text{ m/s}$$



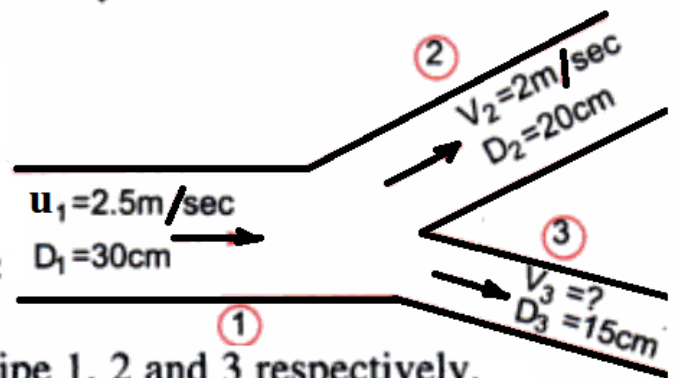
**Example:** A 30 cm diameter pipe, conveying water, branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/s.

**Solution:**

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times .3^2 = 0.07068 \text{ m}^2$$

$$A_2 = \frac{\pi}{4} (.2)^2 = \frac{\pi}{4} \times .4 = 0.0314 \text{ m}^2$$

$$A_3 = \frac{\pi}{4} (.15)^2 = \frac{\pi}{4} \times 0.225 = 0.01767 \text{ m}^2$$



Let  $Q_1, Q_2$  and  $Q_3$  are discharges in pipe 1, 2 and 3 respectively.

Then according to continuity equation  $Q_1 = Q_2 + Q_3$

$$Q_1 = A_1 u_1 = 0.07068 \times 2.5 \text{ m}^3/\text{s} = 0.1767 \text{ m}^3/\text{s}$$

$$Q_2 = A_2 u_2 = .0314 \times 2.0 = .0628 \text{ m}^3/\text{s}$$

$$0.1767 = 0.0628 + Q_3 \implies \therefore Q_3 = 0.1139 \text{ m}^3/\text{s}$$

$$\text{But } Q_3 = A_3 \times u_3 = .01767 \times u_3 = .1139$$

$$\therefore u_3 = \frac{.1139}{.01767} = 6.44 \text{ m/s}$$

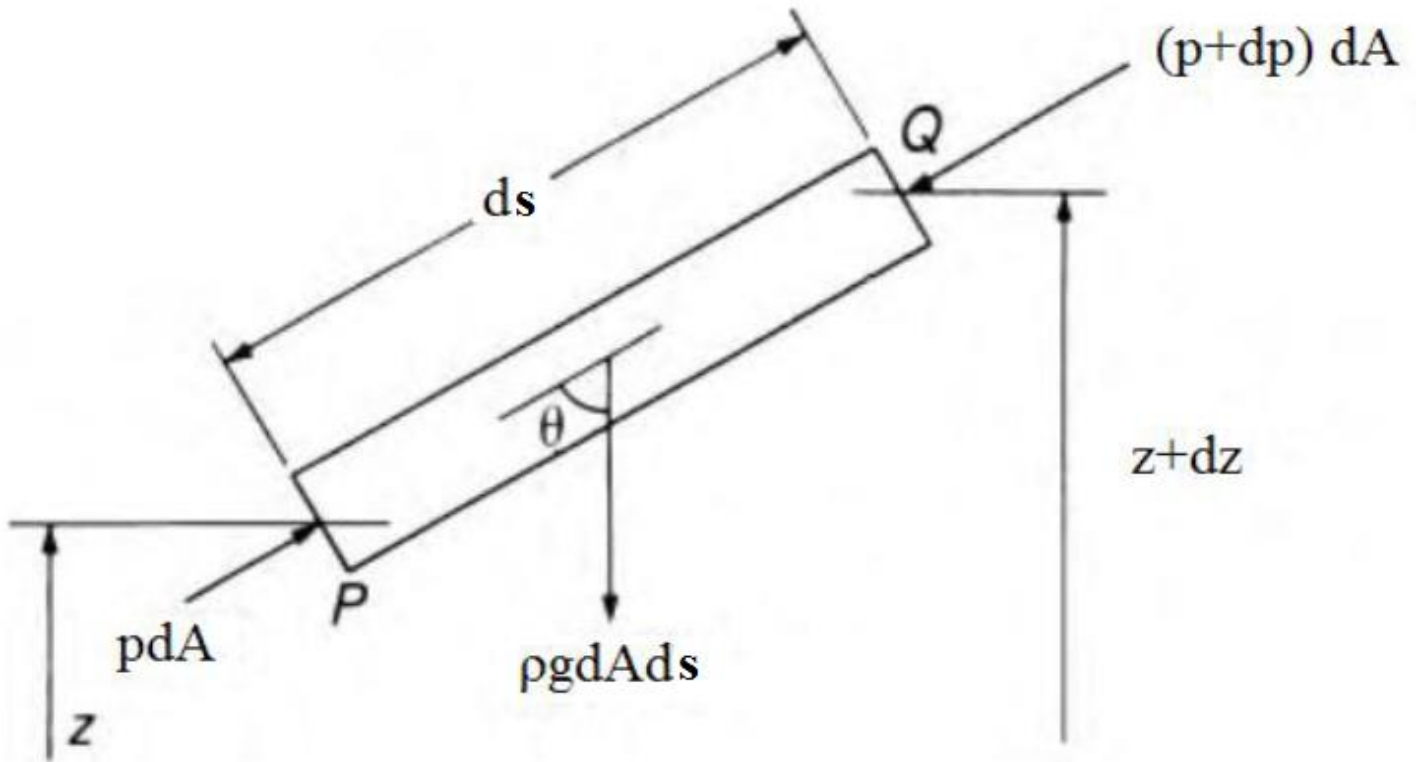
**ENERGY EQUATION:**

This is equation of motion in which the forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of a fluid element along a stream-line as:

Consider a stream-line in which flow is taking place in s-direction as shown in figure. Consider a cylindrical element of cross-section  $dA$  and length  $ds$ . The forces acting on the cylindrical element are:

1. Pressure force  $p dA$  in the direction of flow.
2. Pressure force  $\left( p + \frac{\partial p}{\partial s} ds \right) dA$  opposite to the direction of flow.
3. Weight of element  $\rho g dA ds$ .

Let  $\theta$  is the angle between the direction of flow and the line of action of the weight of element.



The resultant force on the fluid element in the direction of s must be equal to the mass of fluid element x acceleration in the s direction.

$$p dA - \left( p + \frac{\partial p}{\partial s} ds \right) dA - \rho g dA ds \cos \theta = \rho dA ds * a_s \text{ -----(1)}$$

where  $a_s$  is the acceleration in the direction of S.

Now  $a_s = du/dt$  where u is a function of s and t.

$$a_s = \frac{\partial u}{\partial s} \frac{ds}{dt} + \frac{\partial u}{\partial t} = \frac{u \partial u}{\partial s} + \frac{\partial u}{\partial t} \quad \left\{ \frac{ds}{dt} = u \right\}$$

If the flow is steady,  $\frac{\partial u}{\partial t} = 0$ ,  $a_s = \frac{u \partial u}{\partial s}$

Substituting the value of  $a_s$  in equation (1) and simplifying the equation, we get

$$-\frac{\partial p}{\partial s} ds dA - \rho g dA ds \cos \theta = \rho dA ds * \frac{u \partial u}{\partial s} \quad , \quad \text{Dividing by } \rho ds dA, \quad -\frac{\partial p}{\rho \partial s} - g \cos \theta = \frac{u \partial u}{\partial s}$$

$$\frac{\partial p}{\rho \partial s} + g \cos \theta + \frac{u \partial u}{\partial s} = 0 \quad , \quad \text{But from the figure } \cos \theta = \frac{dz}{ds}$$

$$\frac{1}{\rho} \frac{\partial p}{\partial s} + g \frac{dz}{ds} + \frac{u \partial u}{\partial s} = 0 \quad \text{or} \quad \frac{\partial p}{\rho} + g dz + u du = 0, \quad \frac{\partial p}{\rho} + g dz + u du = 0 \text{ -----(2)}$$

The above equation is known as Euler's equation of motion.

Bernoulli's equation is obtained by integrating the above Euler's equation of motion.

$$\int \frac{\partial p}{\rho} + \int g dz + \int u du = constant$$

If the flow is incompressible,  $\rho$  is a constant and

$$\frac{p}{\rho} + gz + \frac{u^2}{2} = cons \tan t \quad \frac{p}{\rho g} + \frac{u^2}{2g} + z = cons \tan t \quad \text{-----}(3)$$

The above equation is known as Bernoulli's equation.

$\frac{p}{\rho g}$  = pressure energy per unit weight of fluid or pressure Head

$\frac{u^2}{2g}$  = kinetic energy per unit weight or kinetic Head

$z$  = potential energy per unit weight or potential Head

**ASSUMPTIONS:**

The following are the assumptions made in the derivation of Bernoulli's equation:

- (i)The fluid is ideal, i.e. viscosity is zero
- (ii)The flow is steady
- (iii)The flow is incompressible
- (iv)The flow is irrotational

**Statement of Bernoulli's Theorem:**

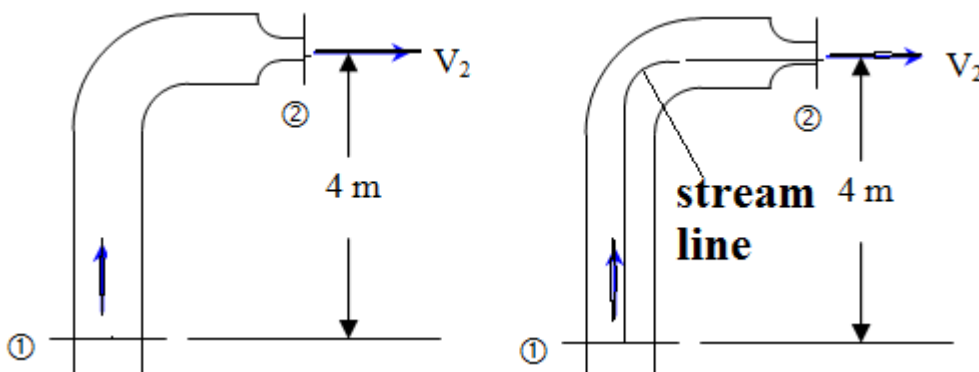
It states in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant. The total energy consists of pressure energy, kinetic energy and potential energy or datum energy. These energies per unit weight of the fluid are:

Pressure energy =  $\frac{p}{\rho g}$  ,    Kinetic energy =  $\frac{u^2}{2g}$  ,    Datum energy =  $z$

Thus mathematically, Bernoulli's theorem is written as

$$\frac{p}{\rho g} + \frac{u^2}{2g} + z = con \tan t$$

Example: Water flows steadily up the vertical 0.1 m diameter pipe and out the nozzle, which is 0.05 m in diameter, discharging to atmospheric pressure. The stream velocity at the nozzle exit must be 20 m/s. Calculate the minimum gage pressure required at section ①.



Assumptions: Steady state condition, Incompressible fluid flow (working fluid, water, is usually assumed to be incompressible), Inviscid fluid flow, The flow is irrotational.

Solution: applying continuity equation between 1 and 2 we have  $u_1 A_1 = u_2 A_2$  and  $u_1 = u_2 A_2 / A_1$

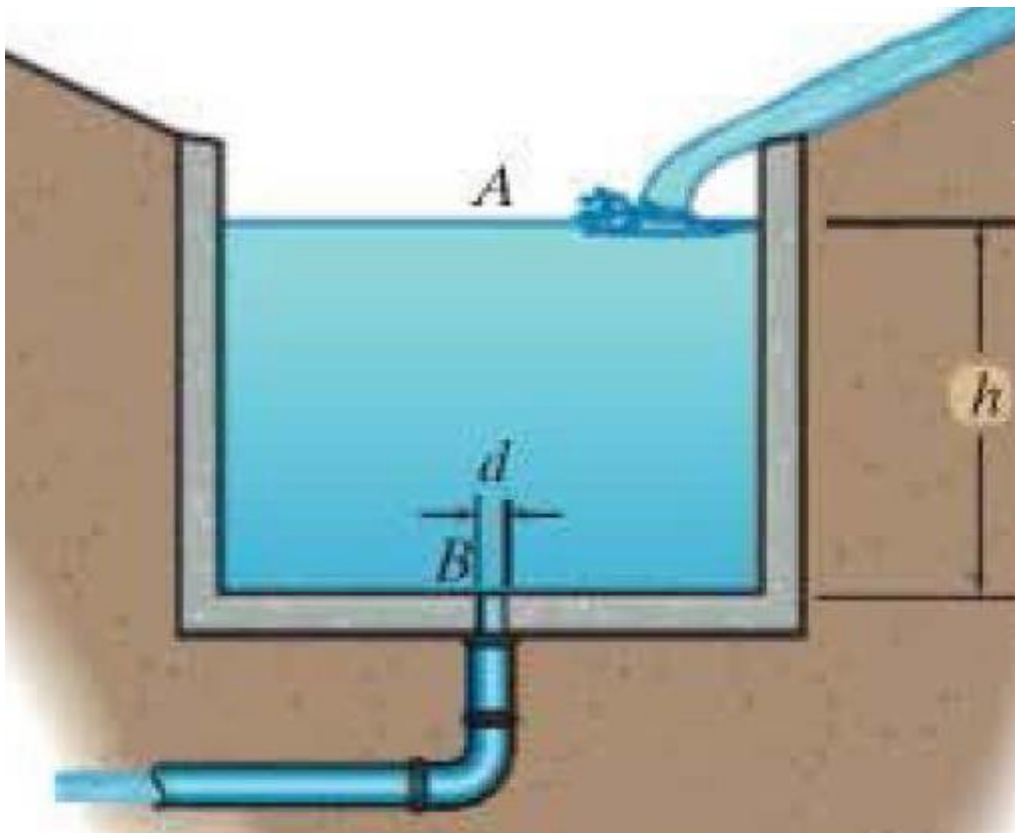
The Bernoulli's equation can be applied between any two points on a streamline provided that the all assumptions are satisfied. The result is

$$\frac{p_1}{\rho} + \frac{u_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{u_2^2}{2} + gz_2 \quad \text{and} \quad \frac{p_1 - p_2}{\rho} = \frac{1}{2}[u_2^2 - u_1^2] + g(z_2 - z_1) \quad \text{also}$$

$$\frac{p_1 - p_2}{\rho} = \frac{1}{2}\left[u_2^2 - \left(\frac{A_2}{A_1}u_2\right)^2\right] + g(z_2 - z_1) \quad \text{and finally} \quad p_{1\text{gage}} - p_{2\text{gage}} = \rho\left\{\frac{1}{2}u_2^2\left[1 - \left(\frac{D_2}{D_1}\right)^2\right] + g(z_2 - z_1)\right\}$$

$$p_{1\text{gage}} - (0) = (1000)\left\{\frac{1}{2}(20)^2\left[1 - \left(\frac{(0.05)^2}{(0.1)^2}\right)^2\right] + (9.81)[(4\text{m}) - (0)]\right\} = 226500\text{Pa} = 226.5\text{kPa}$$

**Example:** Water is discharged through the drain pipe at *B* from the large basin حوض at 0.03 m<sup>3</sup>/s. If the diameter of the drain pipe is *d* = 60 mm, determine the pressure at *B* just inside the drain when the depth of the water is *h* = 2 m.



Solution:

$$Q_B = u_B A_B \Rightarrow u_B = Q_B / A_B = 0.03 / \left(\pi * \frac{0.06^2}{4}\right) = 10.61 \text{ m/s}$$

The Bernoulli's equation can be applied between A and B

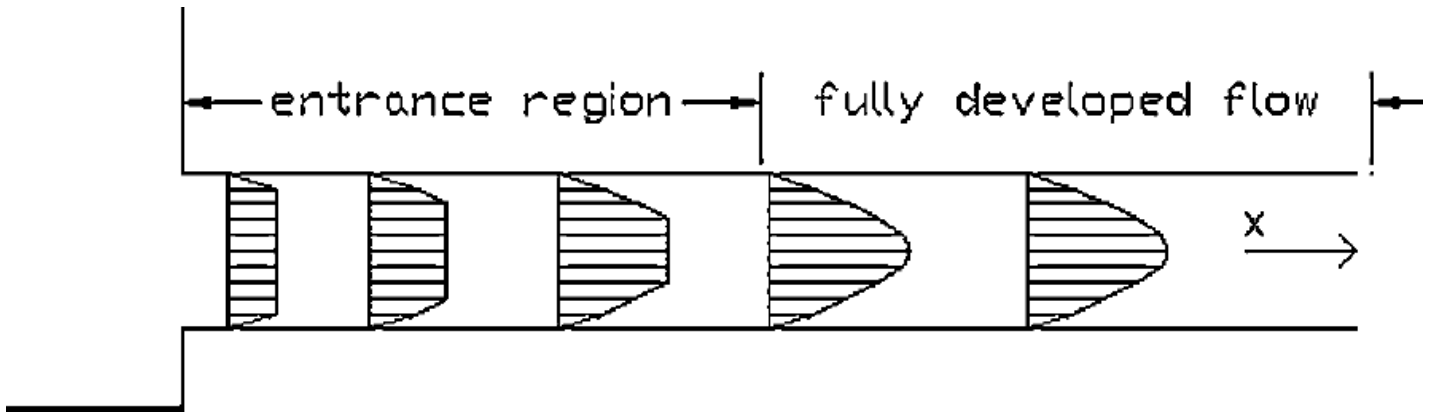
$$\frac{p_A}{\rho} + \frac{u_A^2}{2} + gz_A = \frac{p_B}{\rho} + \frac{u_B^2}{2} + gz_B, \quad \text{but } p_A = p_{\text{atm}} \quad \text{and assume } u_A = 0 \quad (\text{large basin})$$

$$\frac{p_A - p_B}{\rho} = \frac{u_B^2}{2} + g(z_B - z_A) \Rightarrow \frac{p_{\text{atm}} - p_{\text{atm}} - p_{B,\text{gage}}}{\rho} = \frac{u_B^2}{2} + g(z_B - z_A)$$

$$\begin{aligned} p_{B,\text{gage}} &= \rho\left\{-\frac{u_B^2}{2} + g(z_A - z_B)\right\} = 10^3(-56.3 + 9.81 * 2) = -36666\text{Pa} \\ &= -36.666 \text{ kPa} \end{aligned}$$

## Fluid Flow Through pipes

The fluid velocity is shown across the pipe cross section and at various points along the length of the pipe. In the entrance region, the flow begins with a relatively flat velocity profile and then develops an increasingly parabolic flow profile as the distance  $x$  along the pipe increases. Once the flow profile becomes constant and no longer changes with increasing  $x$ , the velocity profile does not change. The region in which the velocity profile is constant is known as the region of fully developed flow.



### FLOW OF VISCOUS FLUID THROUGH CIRCULAR PIPE:

For the flow of viscous fluid through circular pipe, the velocity distribution across a section, the ratio of maximum velocity to average velocity, the shear stress distribution and drop of pressure for a given length is to be determined. The flow through circular pipe will be viscous or laminar, if the Reynold's number is less than 2000. The expression

for Reynold's number is given by  $R_e = \frac{\rho u d}{\mu}$

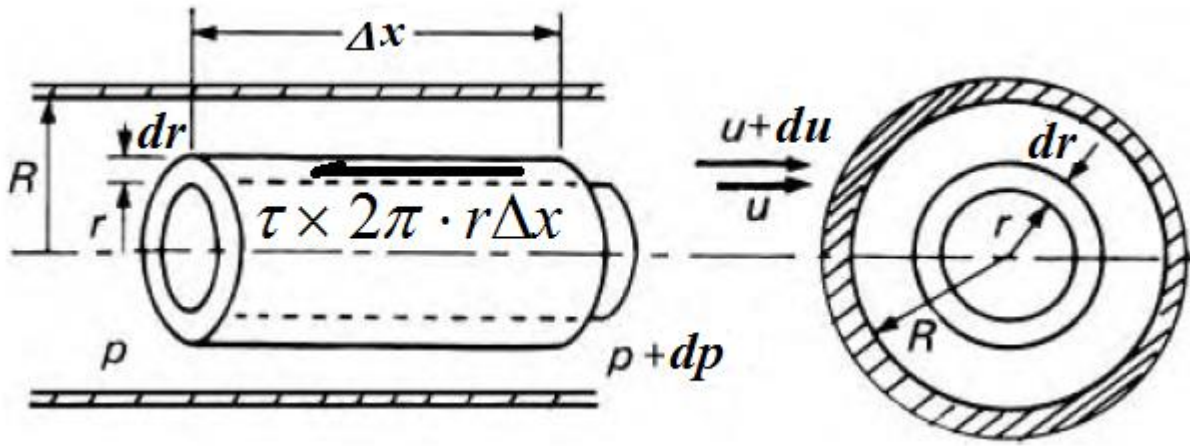
#### Poiseuille's law (Velocity as a function of radius)

This law describes steady, laminar, incompressible, and viscous flow of a Newtonian fluid in a rigid, cylindrical tube of constant cross section. The law was published by Poiseuille in 1840.

Consider a horizontal pipe of radius  $R$ . The viscous fluid is flowing from left to right in the pipe as shown in figure. Consider a fluid element of radius  $r$ , sliding in a cylindrical fluid element of radius  $(r+dr)$ . Let the length of fluid element be  $\Delta x$ . If 'p' is the intensity of pressure on the face AB, then the intensity of pressure on the face CD will be

$\left( p + \frac{\partial p}{\partial x} \Delta x \right)$ . The forces acting on the fluid element are:





1. The pressure force,  $p \times \pi \cdot r^2$  on the left face
2. The pressure force  $\left( p + \frac{\partial p}{\partial x} \Delta x \right) \cdot \pi \cdot r^2$  on the right face
3. The shear force,  $\tau \times 2\pi \cdot r \Delta x$  on the surface of fluid element. As there is no acceleration, hence the summation of all forces in the direction of flow must be zero.

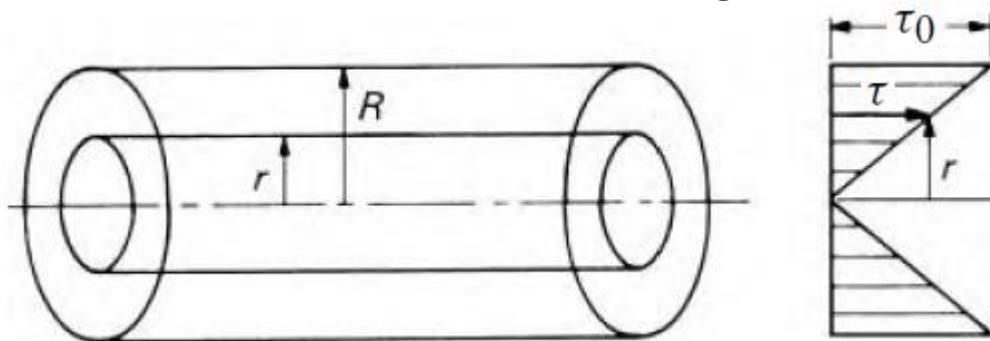
$$p \pi \cdot r^2 - \left( p + \frac{\partial p}{\partial x} \Delta x \right) \cdot \pi r^2 - \tau \times 2\pi \cdot r \Delta x = 0$$

$$-\frac{\partial p}{\partial x} \Delta x \pi \cdot r^2 - \tau \times 2\pi \cdot r \Delta x = 0$$

$$-\frac{\partial p}{\partial x} r - 2\tau = 0$$

$$\tau = -\frac{\partial p}{\partial x} \frac{r}{2} \text{ -----(1)}$$

The shear stress  $\tau$  across a section varies with 'r' as  $\frac{\partial p}{\partial x}$  across a section is constant. Hence shear stress across a section is linear as shown in figure.



(i) velocity Distribution: To obtain the velocity distribution across a section, the value of shear stress  $\tau = \mu \frac{\partial u}{\partial y}$  is substituted in equation (1)

But in the relation  $\tau = \mu \frac{\partial u}{\partial y}$ ,  $y$  is measured from the pipe wall. Hence

$$y = R - r \quad \text{and} \quad dy = -dr$$

$$\tau = \mu \frac{\partial u}{-\partial r} = -\mu \frac{du}{dr}$$

substituting this value in equation (1)

$$-\mu \frac{du}{dr} = -\frac{\partial p}{\partial x} \frac{r}{2}$$

$$\frac{du}{dr} = \frac{1}{2\mu} \frac{\partial p}{\partial x} r$$

Integrating the equation w.r.t 'r' we get

$$u = \frac{1}{4\mu} \frac{\partial p}{\partial x} r^2 + C \quad \text{-----(2)}$$

where C is the constant of integration and its value is obtained from the boundary condition that at  $r=R, u=0$

$$0 = \frac{1}{4\mu} \frac{\partial p}{\partial x} R^2 + C$$

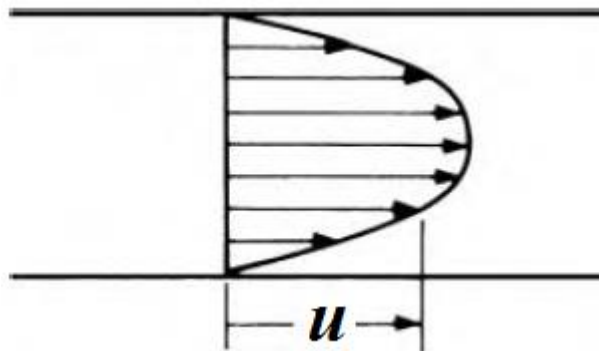
$$C = -\frac{1}{4\mu} \frac{\partial p}{\partial x} R^2$$

Substituting this value of C in equation (2), we get

$$u = \frac{1}{4\mu} \frac{\partial p}{\partial x} r^2 - \frac{1}{4\mu} \frac{\partial p}{\partial x} R^2$$

$$u = \frac{1}{4\mu} \frac{\partial p}{\partial x} [R^2 - r^2] \quad \text{-----(3)}$$

In equation (3) values of  $\mu$ ,  $\frac{\partial p}{\partial x}$  and  $r$  are constant, which means the velocity  $u$ , varies with the square of  $r$ . Thus the equation (3) is an equation of parabola. This shows that the velocity distribution across the section of a pipe is parabolic. This velocity distribution is shown in fig. below.



**(ii) Ratio of Maximum velocity to average velocity:**

The velocity is maximum, when  $r = 0$  in equation (3). Thus maximum velocity,  $U_{max}$  is obtained as

$$U_{\max} = -\frac{1}{4\mu} \frac{\partial p}{\partial x} R^2 \text{-----(4)}$$

The average velocity,  $\bar{u}$ , is obtained by dividing the discharge of the fluid across the section by the area of the pipe ( $\pi R^2$ ). The discharge (Q) across the section is obtained by considering the through a ring element of radius r and thickness the as shown in fig(b). The fluid flowing per second through the elementary ring

$dQ = \text{velocity at a radius } r * \text{area of ring element}$

$$= \mathbf{u} * 2\pi r dr$$

$$= -\frac{1}{4\mu} \frac{\partial p}{\partial x} [R^2 - r^2] * 2\pi r dr$$

$$\begin{aligned} Q = \int_0^R dQ &= \int_0^R -\frac{1}{4\mu} \frac{\partial p}{\partial x} [R^2 - r^2] * 2\pi r dr = \frac{1}{4\mu} \left( -\frac{\partial p}{\partial x} \right) * 2\pi \int_0^R (R^2 - r^2) r dr \\ &= \frac{1}{4\mu} \left( -\frac{\partial p}{\partial x} \right) * 2\pi \int_0^R (R^2 r - r^3) dr = \frac{1}{4\mu} \left( -\frac{\partial p}{\partial x} \right) * 2\pi \left[ \frac{R^2 r^2}{2} - \frac{r^4}{4} \right] \\ &= \frac{1}{4\mu} \left( -\frac{\partial p}{\partial x} \right) * 2\pi \left[ \frac{R^4}{2} - \frac{R^4}{4} \right] = \frac{1}{4\mu} \left( -\frac{\partial p}{\partial x} \right) * 2\pi \left[ \frac{R^4}{4} \right] = \frac{\pi}{8\mu} \left( -\frac{\partial p}{\partial x} \right) * 2\pi R^4 \end{aligned}$$

$$\text{Average velocity, } \bar{u} = \frac{Q}{\text{Area}} = \frac{\frac{\pi}{8\mu} \left( -\frac{\partial p}{\partial x} \right) R^4}{\pi R^2}$$

$$\bar{u} = \frac{1}{8\mu} \left( -\frac{\partial p}{\partial x} \right) R^2 \text{-----(5)}$$

Dividing equation (4) by equation (5)

$$\frac{U_{\max}}{\bar{u}} = \frac{\frac{1}{4\mu} \frac{\partial p}{\partial x} R^2}{\frac{1}{8\mu} \left( -\frac{\partial p}{\partial x} \right) R^2} = 2.0$$

Ratio of maximum velocity to average velocity = 2.0

**(iii) Drop of pressure for a given length (L) of a pipe:**

From equation (5), we have

$$\bar{u} = \frac{1}{8\mu} \left( -\frac{\partial p}{\partial x} \right) R^2 \quad \text{or} \quad \left( \frac{-\partial p}{\partial x} \right) = \frac{8\mu \bar{u}}{R^2}$$

Integrating the above equation w.r.t . x, we get

$$-\int_2^1 dp = \int_2^1 \frac{8\mu \bar{u}}{R^2} dx$$

$$\begin{aligned}
 -[p_1 - p_2] &= \frac{8\mu\bar{u}}{R^2} [x_1 - x_2] \\
 [p_1 - p_2] &= \frac{8\mu\bar{u}}{R^2} [x_2 - x_1] \\
 &= \frac{8\mu\bar{u}}{R^2} L \quad \{x_2 - x_1 = L \text{ from equation (3)}\} \\
 &= \frac{8\mu\bar{u}L}{\left(\frac{D}{2}\right)^2} \quad \{R = \frac{D}{2}\} \\
 [p_1 - p_2] &= \frac{32\mu\bar{u}L}{D^2}, \quad \text{where } p_1 - p_2 \text{ is the drop of pressure}
 \end{aligned}$$

$$\text{Loss of pressure head} = \frac{p_1 - p_2}{\rho g}$$

$$\frac{p_1 - p_2}{\rho g} = h_L = \frac{32\mu\bar{u}L}{\rho g D^2} \text{-----(6)}$$

Equation (6) is called Hagen Poiseuille Formula.

**Ex1:** Show that for the Poiseuille flow in a tube of radius  $R$ , the wall shearing stress can be obtained from the relationship:

$$\tau_w = \frac{4\mu Q}{\pi R^3}$$

for a Newtonian fluid of viscosity  $\mu$ . The volume rate of flow is  $Q$ .

**EX2:** Determine the wall shearing stress for a fluid, having a viscosity of 3.5 cP, flowing with an average velocity of 9 cm/s in a 3-mm-diameter tube. What is the corresponding Reynolds number? (The fluid density  $\rho = 1.06 \text{ g/cm}^3$ .)

**EX: In a 5-mm-diameter vessel, what is the value of the flow rate that causes a wall shear stress of  $0.84 \text{ N/m}^2$ ? Would the corresponding flow be laminar or turbulent?**

<https://drive.google.com/drive/folders/1rnueupAiGlJnUGfxeP621XH213CM73Zg>