Fluids

Definition of fluid: A fluid is defined as a substance that deforms continuously under application of a shearing stress, regardless of how small the stress is.

To study the behavior of materials that act as fluids, it is useful to define a number of important fluid properties, which include density, specific weight, specific gravity, and viscosity.

Density is defined as the mass per unit volume of a substance and is denoted by the Greek character ρ (rho). The SI units for ρ are kg/m^3.

Specific weight is defined as the weight per unit volume of a substance. The SI units for specific weight are N/m^3.

Specific gravity ***S*** is the ratio of the weight of a liquid at a standard reference temperature to the weight of water. For example, the specific gravity of mercury $S_{Hg} = 13.6$ at 20^oC. Specific gravity is a unit-less parameter.

Density and specific weight are measures of the "heaviness" of a fluid.

Example: What is the specific gravity of human blood, if the density of blood is 1060 kg/m3?

Solution: $S_{blood} = {\rho_{blood}}/{\rho_{water}} = {1060}/{1000} = 1.06$

Viscosity, shearing stress and shearing strain

Viscosity is a measure of a fluid's resistance to flow. It describes the internal friction of a moving fluid. A fluid with large viscosity resists motion because its molecular makeup gives it a lot of internal friction. A fluid with low viscosity flows easily because its molecular makeup results in very little friction when it is in motion. Gases also have viscosity, although it is a little harder to notice it in ordinary circumstances.

To understand viscosity, let fluid between two parallel infinite in width and length plates. See Fig. 1.1. The bottom plate A is fixed and the upper plate B is moveable. The vertical distance between the two plates is represented by *h*. A constant force *F* is applied to the moveable plate B causing it to move along at a constant velocity u_B with respect to the fixed plate. This behavior is consistent with the definition of a fluid: a material that deforms continuously under the application of a **shearing stress**, regardless of how small the stress is.

After some infinitesimal time *dt*, a line of fluid that was vertical at time $t= 0$ will move to a new position, as shown by the dashed line in Fig. 1.1. The tan of angle between the line of fluid at $t = 0$ and $t = t + dt$ is defined as the **shearing strain *du/dy***.

The fluid that touches plate A has zero velocity $u=0$. The fluid that touches plate B moves with the same velocity as that of plate B, u_B. That is, the molecules of the fluid adhere to the plate and do not slide along its surface. This is known as the no-slip condition. The **no-slip condition** is important in fluid mechanics. All fluids, including both gasses and liquids, satisfy this condition.

Let the distance from the fixed plate A to some arbitrary point above the plate be *y*. The velocity *u* of the fluid between the plates is a function of the distance above the fixed plate A, or $u=u(y)$. Let us define the velocity gradient as the change in fluid velocity with respect to *y*.

$$Velocity\ gradient \equiv du/dy$$

Note that the velocity gradient is represented the time rate of shearing strain.

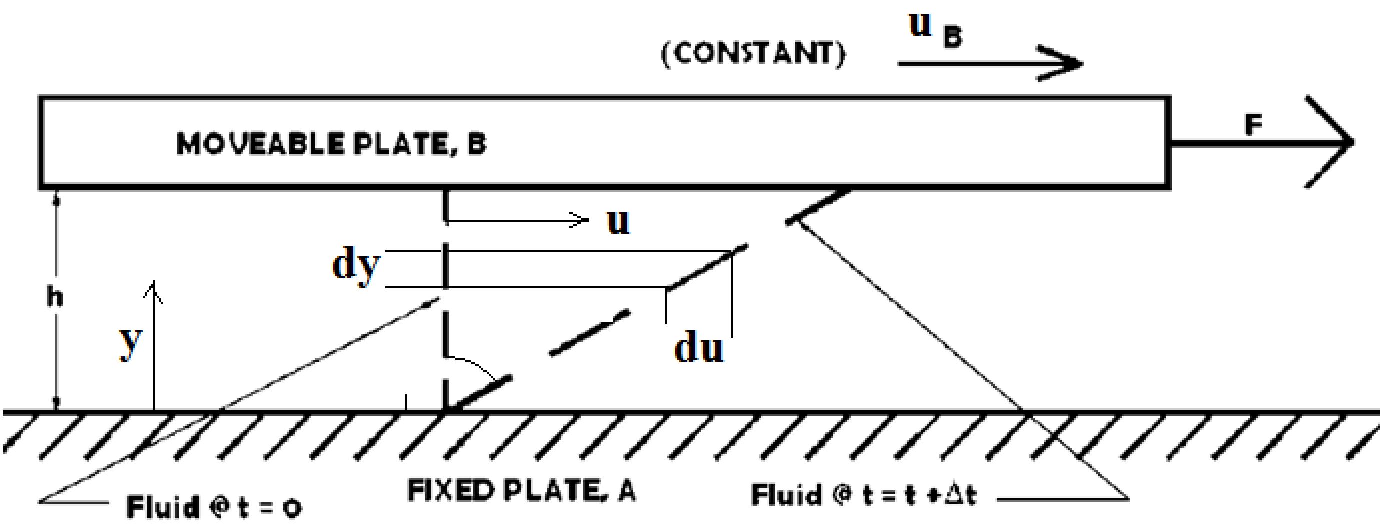


Figure 1.1 Velocity profile in a fluid between two parallel plates.

If the velocity of the fluid at any point between the plates varies linearly between $u= 0$ and $u = u_B$, the velocity gradient can also be written as

$$Velocity\ gradient = u_B/h$$

The velocity profile is a graphical representation of the velocity gradient.

Figure 1.2 represents the shear stress on an element of the fluid at some arbitrary point between the plates in Figs. 1.1. The shear stress on the top of the element results in a force that pulls the element "downstream." The shear stress at the bottom of the element resists that movement. Since the fluid element shown will be moving at a constant velocity, and will not be rotating, the shear stress on the element τ must be the same as the shear stress τ. Therefore, $\frac{d\tau}{dy} = 0$ *and* $\tau_A = \tau_B = \tau_{wall}$

Physically, the shearing stress at the wall may also be represented by

$$\tau_A = \tau_B = {}^{force}/_{plate\ area} = {}^{F}/_{A_{parallel}}$$

The shear stress on a fluid is related to the rate of shearing strain. $\tau \propto du/dy$

In fact, the relationship between shearing stress and rate of shearing strain is determined by the fluid property known as **viscosity** sometimes referred to by the name (**absolute viscosity or dynamic viscosity**) represented by the Greek letter **μ** (mu).

$$\tau = \mu * du/dy$$

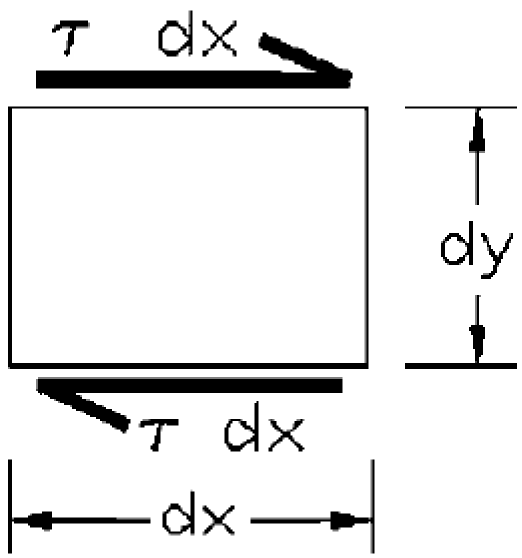


Figure 1.2 Shear stress on an element of the fluid in figure 1.1.

<u>Kinematic viscosity</u> is another fluid property that has been used to characterize flow. It is the ratio of absolute viscosity to fluid density and is represented by the Greek character ν (nu). Kinematic viscosity can be defined by the equation:

$$\nu = \mu/\rho$$

SI unit for absolute viscosity is N.s/m^2 or Pa·s or poise = g/s·cm , and for kinematic viscosity is m^2/s or Stoke = cm^2/s.

<u>Example</u>: The 100-kg plate in Fig. l-3a is resting on a very thin film of oil, which has a viscosity of μ = 0.0652 N s/m^2. Determine the force P that must be applied to the center of the plate to slide it over the oil with a constant velocity of 0.2 m/s. Assume the oil thickness is 0.1 mm, and the velocity profile across this thickness is linear. The bottom of the plate has a contact area of 0.75 m^2 with the oil.

<u>SOLUTION</u>

<u>Assumption</u>: The oil is a Newtonian fluid, and so Newtonian's law of viscosity can be applied.

<u>Analysis</u>: First we draw the free-body diagram of the plate in order to relate the shear force F caused by the oil on the bottom of the plate to the applied force P, Fig.1-3b. Because the plate moves with constant velocity, the force equation of equilibrium in the horizontal direction applies.

$$\sum F_x = 0 \qquad F - P\cos 30° = 0 \;\rightarrow\; F = 0.8660P$$

The effect of this force *on the oil* is in the opposite direction, and so the *shear stress* on the top of the oil acts to the left. It is

$$\tau = \frac{F}{A} = \frac{0.8660P}{0.75} = 1.155P$$

Since the velocity profile is assumed to be linear, Fig. 1- 3c, the velocity gradient is constant *du/dy* = *U/h*, and so

$$\tau = \mu \frac{du}{dy} = \mu \frac{U}{h} \quad \Longrightarrow \quad 1.155P = 0.0652 * \left(\frac{0.2}{0.0001}\right) \quad \Longrightarrow \quad P = 113N$$

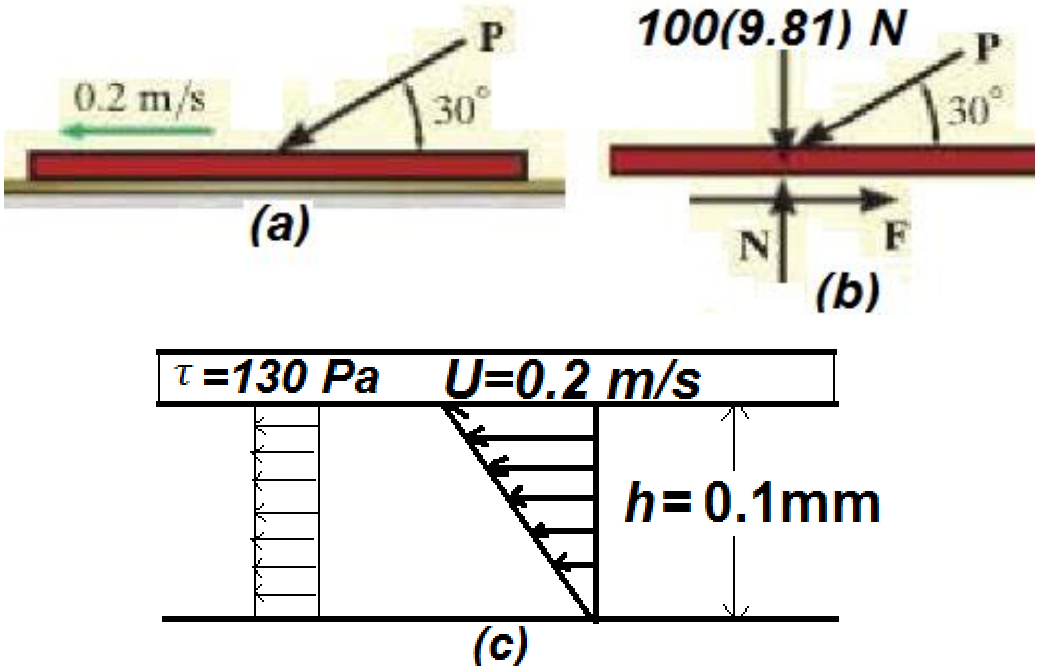


Figure 1.3 plate is sliding on a very thin film of oil

Example: The plate in Fig.1.4 rests on top of the thin film of water; the viscosity of water at a temperature of 25°C is μ = 0.897(10^{-3}) N · s/m^2. When a small force F is applied to the plate, the velocity profile across the thickness of the fluid can be described as *u* = (40y - 800y^2) m/ s. where *y* is in meters. Determine the shear stress acting on the fixed surface and on the bottom of the plate.

Solution:

Assumption: water is a Newtonian fluid, and so Newton's law of viscosity applies.

Analysis: Before applying Newton's law of viscosity, we must first obtain the velocity gradient

$$\frac{du}{dy} = \frac{d}{dy}(40y - 800y^2) = 40 - 1600y$$

Therefore, at the fixed surface, *y* = 0,

$$\tau = \mu \frac{du}{dy}\bigg|_{y\,=\,0} = 0.897 * 10^{-3} * (40 - 0) = 35.88 * 10^{-3}\,\frac{N}{m^2} = 35.88\,mPa$$

And, at the bottom of the moving plate, *y* = 0.01 m,

$$\tau = \mu \frac{du}{dy}\bigg|_{y\,=\,0.01} = 0.897 * 10^{-3} * (40 - 1600(0.01)) = 21.5\,mPa$$

By comparison, the *larger shear stress* develops on the fixed surface rather than on the bottom of the plate since the *velocity gradient* or slope *du/dy* is *large* at the fixed surface. Both of these slopes are indicated by the short dark lines in Fig. 1-4. Also, notice that the equation for the velocity profile must satisfy the boundary condition of no slipping, i.e., at the fixed surface $y = 0$, $u = 0$, and with the movement of the plate at y = 10 mm, $u = U = 0.32$ m/ s.

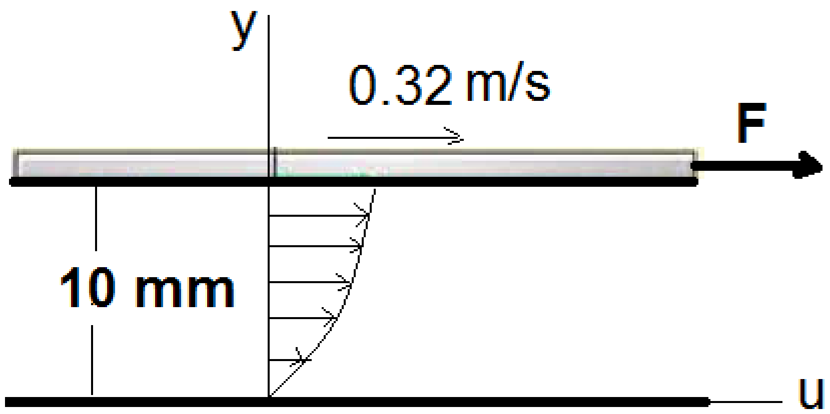


Figure 1.4 plate slides on top of the thin film of water

Newtonian and non-Newtonian Fluids

Newtonian fluids are the fluids of constant viscosity. For common fluids like oil, water, and air, τ and du/dy are linearly related. (see Fig. 1.5a). And the slope of the stress–shearing rate curve is constant represents the viscosity.

For non-Newtonian fluids, τ and du/dy are not linearly related. For those fluids, viscosity can change as a function of the shear rate (rate of shearing strain). Blood is an important example of a non-Newtonian fluid. But, we will investigate the condition under which blood behaves as, and may be considered, a Newtonian fluid.

Shear stress and shear rate are not linearly related for non-Newtonian fluids. Therefore, the slope of the shear stress/shear rate curve is not constant. However, we can still talk about viscosity if we define the **apparent الظاهرية viscosity as the instantaneous الانى slope of the shear stress/shear rate curve.** See Fig. 1.5b.

Shear thinning fluids are non-Newtonian fluids whose apparent viscosity decreases as shear rate increases. Latex paint الصبغات النباتية is a good example of a shear thinning fluid. It is a positive characteristic of the paint that the viscosity is low when one is painting, but that the viscosity becomes higher and the paint sticks to the surface better when no shearing force is present. At low shear rates, blood is also a shear thinning fluid. However, when the shear rate increases above 100 s^{-1}, blood behaves as a Newtonian fluid.

Shear thickening fluids are non-Newtonian fluids whose apparent viscosity increases when the shear rate increases. Quicksand الرمال المتحركة is a good example of a shear thickening fluid. If one tries to move slowly in quicksand, then the viscosity is low and the movement is relatively easy. If one tries to move quickly, then the viscosity increases and the movement is difficult. A mixture of cornstarch دقيق الذرة and water also forms a shear thickening non-Newtonian fluid.

A Bingham plastic is neither a fluid nor a solid. A Bingham plastic can withstand يقاوم a finite shear load and flow like a fluid when that shear stress is exceeded تجاوز الحد. Toothpaste and mayonnaise are examples of Bingham plastics. Blood is also a Bingham plastic and behaves as a solid at shear rates very close to zero. The yield stress for blood is very small, approximately in the range from 0.005 to 0.01 N/m^2.

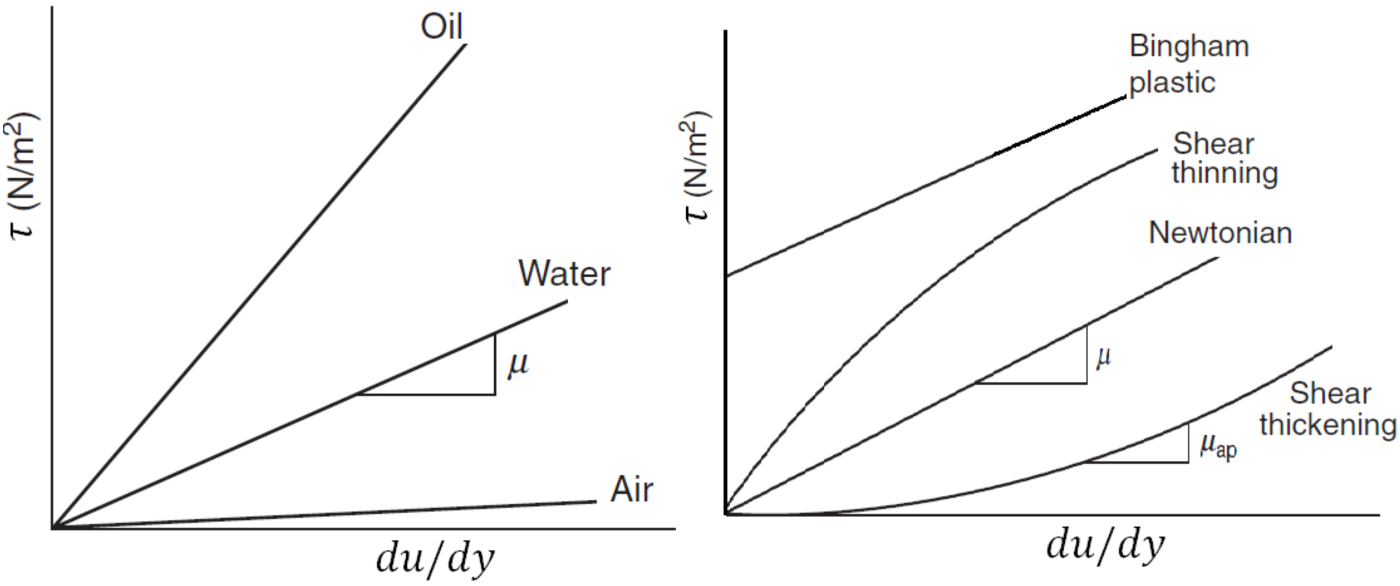


Figure 1.5 relationship between τ and du/dy (a) Newtonian fluid examples, (b) non-Newtonian fluid compared with Newtonian fluid

Example: An experimental test using human blood at T= 30^{o}C indicates that it exerts a shear stress of τ= 0. 15 N/ m^2 on a surface A, where the measured velocity gradient at the surface is 16.8 s^{-1}. Since blood is a non-Newtonian fluid. Determine its *apparent viscosity* at the surface.

Solution: $\mu_{apparent} = {}^{\tau}/_{(du/dy)} = 0.15/16.8 = 8.9 * 10^{-3}\ Pa.s$

Viscosity Measurements

Rotational Viscometers

The viscometer gives the value of the 'dynamic viscosity'. It is based on the principle that the fluid whose viscosity is being measured is sheared between two surfaces. In these viscometers one of the surfaces is stationary and the other is rotated by an external drive and the fluid fills يملأ the space in between بينهما. (See Fig. 1.6). The measurements are conducted by applying either a constant torque and measuring the changes in the speed of rotation or applying a constant speed and measuring the changes in the torque العزم. There are two main types of these viscometers: rotating cylinder and cone-on-plate مخروط على صفيحة viscometers.

$$\boldsymbol{\mu = \frac{\tau}{du/dy}} \quad , \boldsymbol{\tau = \frac{Force}{Area} = \frac{Torque/raduis}{2\pi * raduis * height} = \frac{Torque}{2\pi * raduis^2 * height}} \quad , \boldsymbol{\frac{du}{dy} = \frac{velocity}{r_c - r_b} = \frac{raduis * \omega}{r_c - r_b}}$$

$$\boldsymbol{\therefore \mu = \frac{Torque * (r_c - r_b)}{2\pi\omega * (raduis)^3 * height}}$$

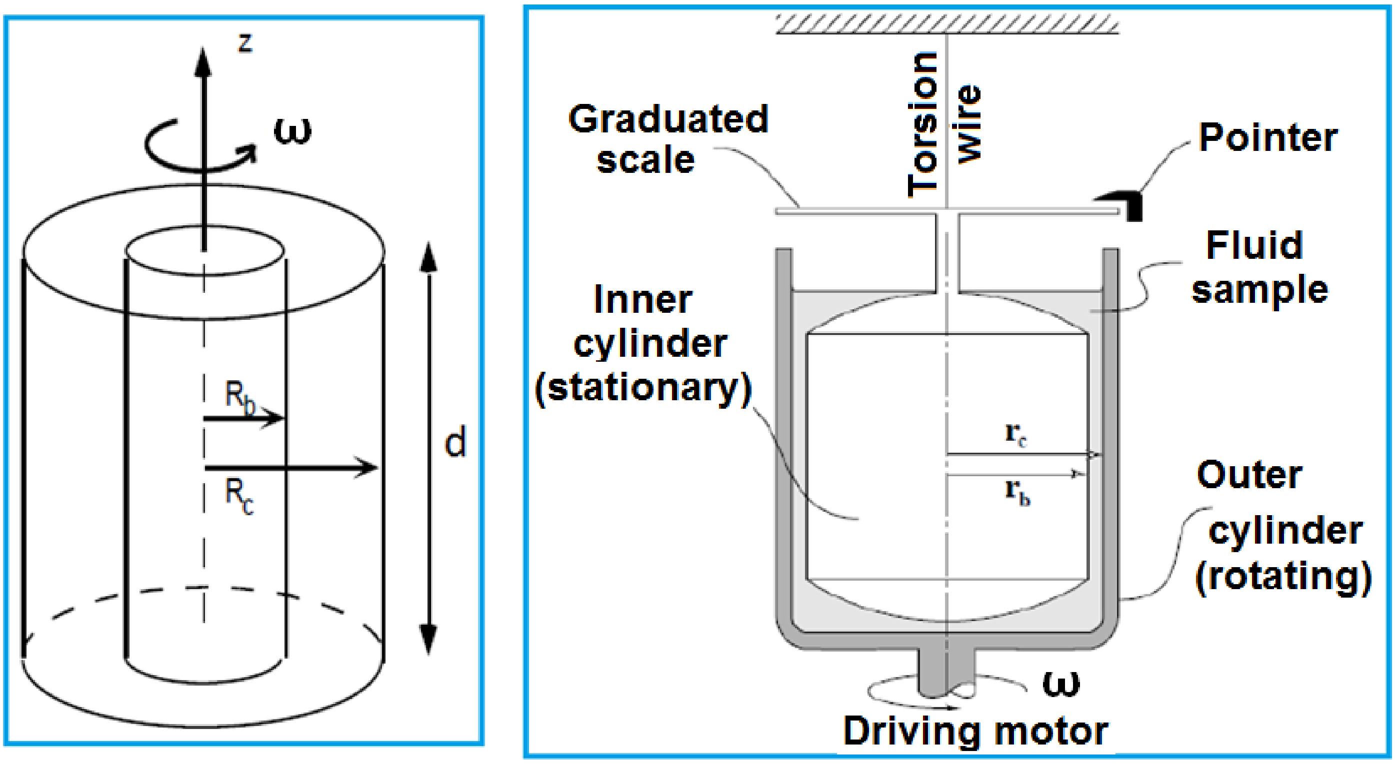
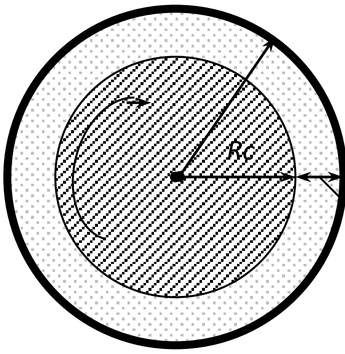


Figure 1.6 Rotational Viscometers

Example: The viscosity of a fluid is to be measured by a viscometer constructed of two 75-mm-long concentric cylinders. The outer diameter of the inner cylinder is 15 cm, and the gap between the two cylinders is 0.12 cm. The inner cylinder is rotated at 200 rpm, and the torque is measured to be 0.8 N-m. Determine the viscosity of the fluid.

Solution The torque and the rpm of a double cylinder viscometer are given. The viscosity of the fluid is to be determined.



Assumptions: **1.** The inner cylinder is completely submerged in oil. **2.** The viscous effects on the two ends of the inner cylinder are negligible. **3.** The fluid is Newtonian.

Analysis Substituting the given values, the viscosity of the fluid is determined to be

$$\mu = \frac{\mathbf{T}(Rd - Rc)}{2\pi . Rc^3 \omega L} = \frac{(0.8\,\mathrm{N\cdot m})(0.0012\,\mathrm{m})}{4\pi^2(0.075\,\mathrm{m})^3(200/60\,\mathrm{s^{-1}})(0.75\,\mathrm{m})} = \mathbf{0.0231}\ Pa.s$$

Compressibility of Fluids

Compressibility β is a measure of the relative volume change of a fluid or solid as a response to a pressure (or mean stress) change.

$$\beta = -\frac{1}{V}\frac{\partial V}{\partial p} \quad or \quad \beta = \frac{1}{\rho}\frac{\partial \rho}{\partial p} \qquad \frac{m^2}{N} \ or\ Pa^{-1}$$

The negative sign in first equation makes the compressibility positive in the (usual) case that an increase in pressure induces a reduction in volume. The variation of volume or density to variation in pressure may be occurs at adiabatic or isothermal process.

Bulk Modulus: is the inverse of the compressibility and often denoted *K*.

$$K = -\frac{V\partial p}{\partial V} \quad \frac{N}{m^2} \quad or \quad Pa$$

Large values of the bulk modulus indicate incompressibility. Incompressibility indicates large pressures are needed to compress the volume slightly. For example, it takes 215 bar to compress water 1% at atmospheric pressure and 15°C. Most liquids are incompressible for most practical engineering problems.

A liquid compressed in a cylinder has a volume of 1000 cm^3 at 1 MN/m^2 and a volume of 995 cm^3 at 2 MN/m^2. What is its bulk modulus of elasticity (K)?

Solution:

$$K = -\frac{\Delta p}{\Delta V/V} = -\frac{2-1}{(995-1000)/1000} = 200\ \text{MPa}$$

Example: **If $K = 2.2$ GPa is the bulk modulus of elasticity for water, what pressure is required to reduce a volume by 0.006 percent?**

Solution:

$$K = -\frac{\Delta p}{\Delta V/V} \qquad 2.2 = -\frac{p_2 - 0}{-0.006} \qquad p_2 = 0.0132\ \text{GPa} \quad \text{or} \quad 13.2\ \text{MPa}$$

Cohesion and Adhesion in Liquids:

Children blow soap bubbles. An underwater spider keeps his air supply in a bubble he carries wrapped around him (الفقاعات التي تطوقه). A technician draws blood into a small-diameter tube just by touching it to a drop on a pricked ثقب finger الاصبع. A premature infant struggles to inflate her lungs (الطفل الخديج يكافح بصعوبة لملئ رئتيه بالاوكسجين). What is the common thread الخيط المشترك? All these activities are dominated by the attractive التجاذب forces between atoms and molecules in liquids both within a liquid and between the liquid and its surroundings.

Attractive forces between molecules of the same type are called cohesive التماسك forces. And Attractive forces between molecules of different types are called adhesive التصاق forces.

Surface Tension

At the interface between a liquid and a gas or two immiscible غير قابل للامتزاج liquids, forces develop forming an analogous متماثل "skin السطح الخارجي" or "membrane غشاء" stretched شد مط ، over the fluid mass which can support weight يسند وزنا.

This "skin" is due to an imbalance of cohesive forces. The interior of the fluid is in balance as molecules of the like fluid are attracting each other while on the interface there is a net inward pulling force.

Cohesive forces between molecules cause the surface of a liquid to contract للانكماش to the smallest possible surface area. This general effect is called surface tension.

Therefore surface tension is the intensity of the molecular attraction per unit length along any line in the surface.

Surface tension is a property of the liquid type, the temperature, and the other fluid at the interface.

Surface tension has the dimension of force per unit length, or of (energy per unit area). The two are equivalent, but when referring to energy per unit of area, it is common to use the term surface energy, which is a more general term in the sense that it applies also to solids.

تربط بين جزيئات المادة المتجانسة قوى تسمى قوى الجذب الجزيئية (قوى التماسك) تعمل على تماسك جزيئات هذه المادة بعضها ببعض، إن قيمة هذه القوى في السوائل تكون أقل مما عليه في الأجسام الصلبة وهذا ما يفسر تغير شكل السائل بتغير الإناء الموجود فيه، بالإضافة إلى تلك القوى توجد قوى تؤثر بين جزيئات السائل وجزيئات الأوساط الأخرى التي تلامسها سواء أكانت حالة تلك الأوساط صلبة أو سائلة أو غازية تدعى هذه القوى ب (قوى التلاصق) بالنسبة للجزيئات الواقعة على سطح السائل.

بالنسبة للجزيئات الواقعة في داخل السائل أي على بعد عدة أقطار جزيئية إلى الأسفل من سطحه، فإن كل جزيء مثل (A) سوف يتأثر بقوى تماسك مع جزيئات السائل الأخرى من جميع الجهات وبنفس القدر تقريباً مما يعني أن جزيء مثل (A) سيكون متأثر بمجموعة متزنة من القوى محصلتها معدومة. أما بالنسبة لجزيئات السائل عند السطح فإن كل جزيء مثل (B) سوف يكون متأثر بقوى تماسك مع جزيئات السائل من الجهة السفلى ومتأثر بقوى التلاصق مع جزيئات الهواء من الجهة العليا وحيث أن كثافة السوائل أكبر بكثير من كثافة الغازات لذلك فإن محصلة هذه القوى تكون في اتجاه قوى التماسك. أي أن كل جزيء عند السطح يكون متأثراً بقوى جذب إلى الداخل (مما يقلل من فرصة شغله موقع سطحي) تؤدي إلى تقلص سطح السائل ليشغل أصغر مساحة ممكنة له. وهذا يفسر الشكل الشبه الكروي لقطرات السائل ويكون عندئذ سطحها أصغر بالنسبة لحجم معين . See Fig. 1.7

(القوى والروابط بين جزيئات السائل هي المسؤلة عن التوتر السطحي . و الجزيئات على السطح ليس لديها جزيئات فوقها لذلك تكون قوى الترابط على الجزيئات الأخرى المحيطة بها أقوى مقارنة بالجزيئات الداخلية).

الآن لنعرف التوتر السطحي (σ_s) لسائل : القوة المؤثرة في وحدة الطول في سطح بزاوية قائمة على أحد جانبي خط مرسوم في السطح. يقاس التوتر السطحي (σ_s) بوحدة.(N/m)

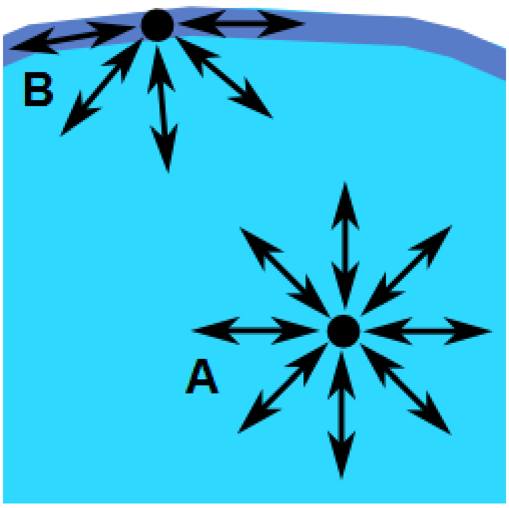


Figure 1.7 forces of surface and internal molecules

<u>Mathematical analysis</u>

The pressure inside a drop of fluid can be calculated using a free-body diagram (Fig.1.8):

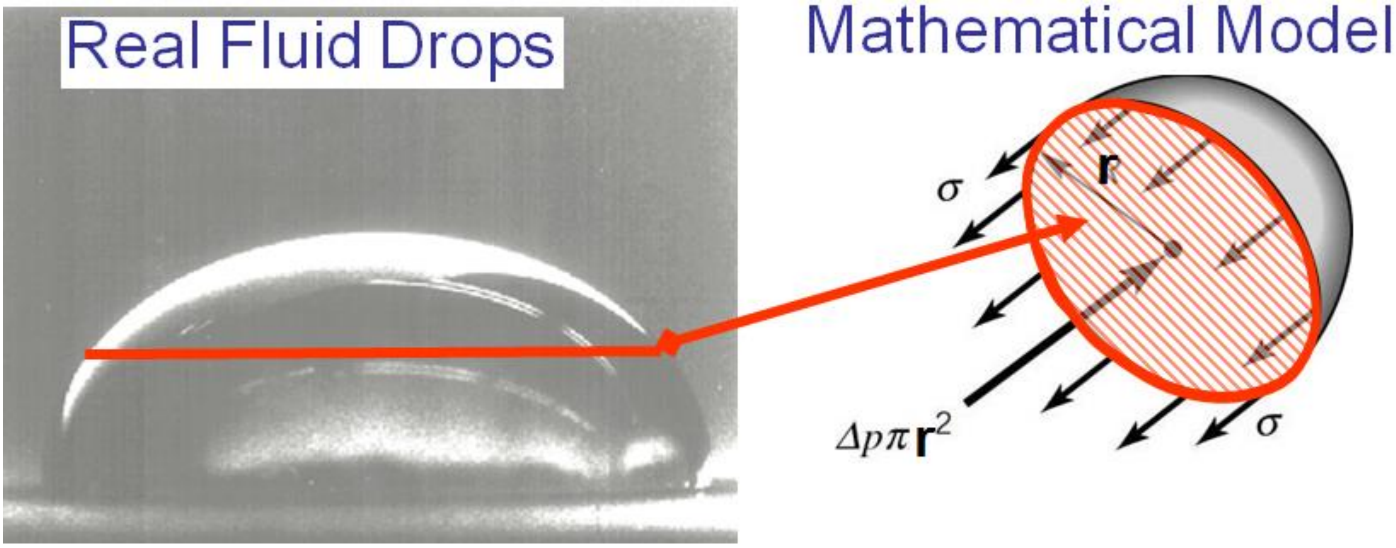


Figure 1.8 forces effect on section of fluid drop

r is the radius of the droplet, σ is the surface tension, Δp is the pressure difference between the inside and outside pressure. The force developed around the edge due to surface tension along the line:

$$F_{surface} = 2\pi \cdot r \sigma$$

Applied to Circumference

This force is balanced by the pressure difference Δp:

$$F_{pressure} = \Delta p \pi r^2$$

Applied to Area

Now, equating the Surface Tension Force to the Pressure Force, we can estimate $\Delta p = p_i - p_e$:

$$\Delta p = \frac{2\sigma}{r}$$

This indicates that the internal pressure in the droplet is greater that the external pressure since the right hand side is entirely positive.

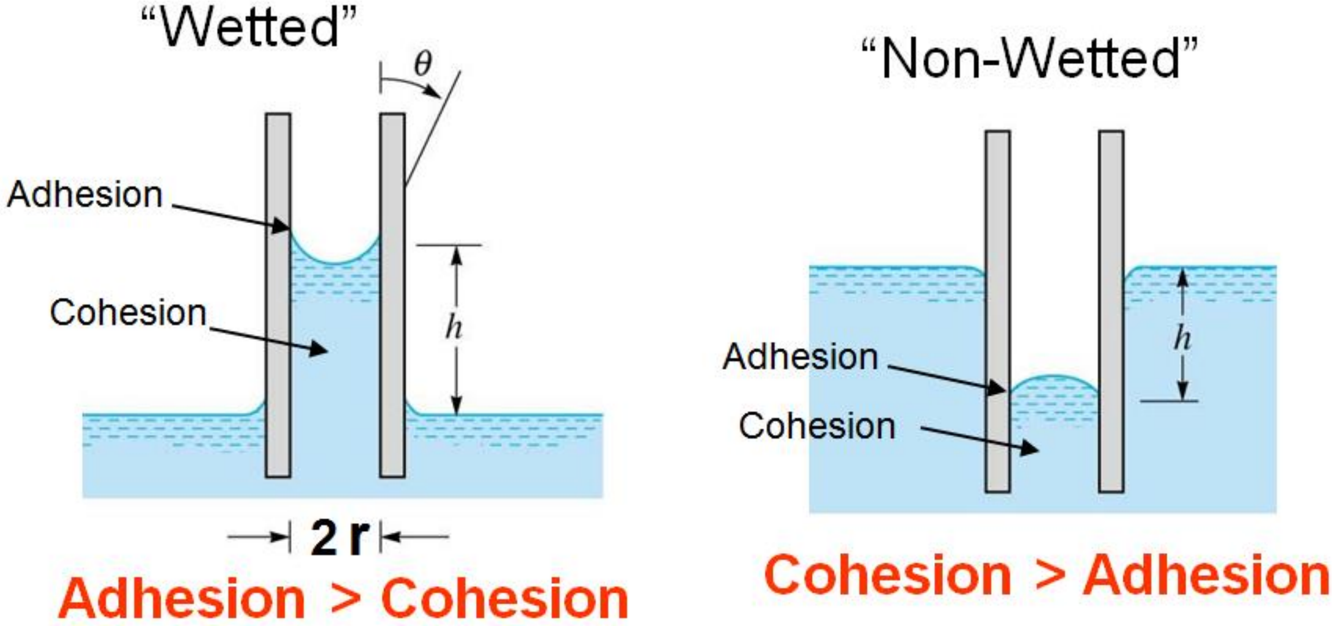
Is the pressure inside a bubble of water greater or less than that of a droplet of water?

Prove the following result:

$$\Delta p = \frac{4\sigma}{r}$$

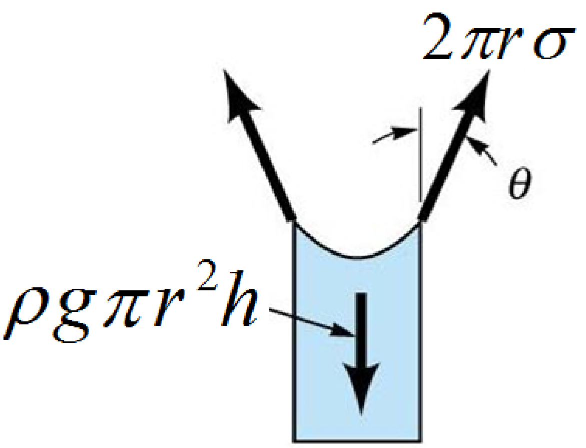
Capillary Action

Capillary action in small tubes which involve a liquid-gas-solid interface is caused by surface tension. The fluid is either drawn up يسحب للاعلى the tube or pushed down يدفع للاسفل.



h is the height, r is the radius of the tube, θ is the angle of contact. The weight of the fluid is balanced with the vertical force caused by surface tension.

Free Body Diagram for Capillary Action for a Wetted Surface:



$$F_{surface} = 2\pi r \sigma \cos\theta$$

$$W = \rho g \pi r^2 h$$

Equating the two and solving for h:

$$h = \frac{2\sigma \cos\theta}{\rho g r}$$

For clean glass in contact with water, θ ≈ 0°, and thus as r decreases, h increases, giving a higher rise.
For a clean glass in contact with Mercury, θ ≈ 130°, and thus h is negative or there is a push down of the fluid.

At what value of contact angle θ does the liquid-solid interface become "non-wetted"? θ > **90°**
Surface tension is apparent in many practical problems such as movement of liquid through soil التربة and other porous media الاوساط المسامية, flow of thin films, formation of drops and bubbles, and the breakup تلاشي of liquid jets.

Example: Given a water-air-glass interface ($\theta \approx 0^\circ$, σ = 0.073 N/m, and ρ = 1000 kg/m^3) with r = 1 mm, determine the capillary height, h.

Solution

$$h = \frac{2(0.073\,N/m)\cos 0^\circ}{(1000\,kg/m^3)(9.81\,m/s^2)(0.001\,m)} = 1.5\,cm$$

Example: For a mercury-air-glass interface with $\theta = 130^\circ$, $\sigma = 0.48$ N/m and $\rho = 13{,}600$ kg/m^3, the capillary rise will be

Solution:

$$h = \frac{2(0.48\,N/m)\cos 130^\circ}{(13{,}600\,kg/m^3)(9.81\,m/s^2)(0.001\,m)} = -0.46\,cm$$

Example:

A small drop of water at *26.6°C* is in contact with the air and has a diameter of *0.508 mm*. If the pressure within the droplet is *565.37 Pa* greater than the atmosphere, what is the value of the surface tension?

Solution: $p(\pi d^2/4) = (\pi d)(\sigma)$, $\sigma = pd/4 = [565.37][0.508*10^{-3}]/4 = 0.0718$ N/m

Pressure is defined as a *normal force exerted by a fluid per unit area*. Units of pressure are N/m^2, which is called a **Pascal** (Pa). Since the unit Pa is too small for pressures encountered in practice, *kilopascal* (1 kPa = 10^3 Pa) and *mega Pascal* (1 MPa = 10^6 Pa) are commonly used. Other units include *bar, atm, kgf/cm^2, lbf/in^2=psi.*

Actual pressure at a given point is called the **absolute pressure**. Most pressure-measuring devices are calibrated to read zero in the atmosphere, and therefore indicate **gage pressure**, $P_{gage}=P_{abs} - P_{atm}$. Pressure below atmospheric pressure are called **vacuum pressure**, $P_{vac}=P_{atm} - P_{abs}$.

Pressure at a Point

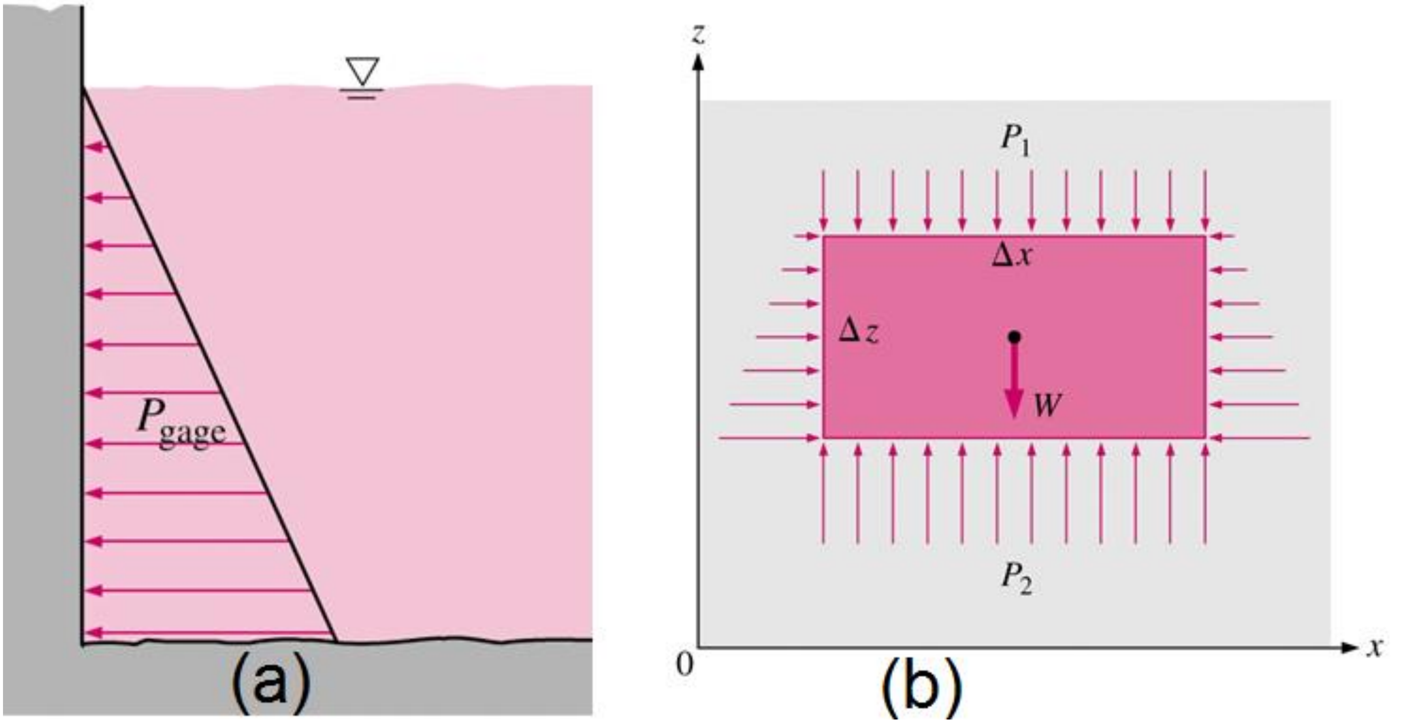
Pressure at any point in a fluid is the same in all directions. Pressure has a magnitude, but not a specific direction, and thus it is a **scalar quantity**.

Variation of Pressure with Depth

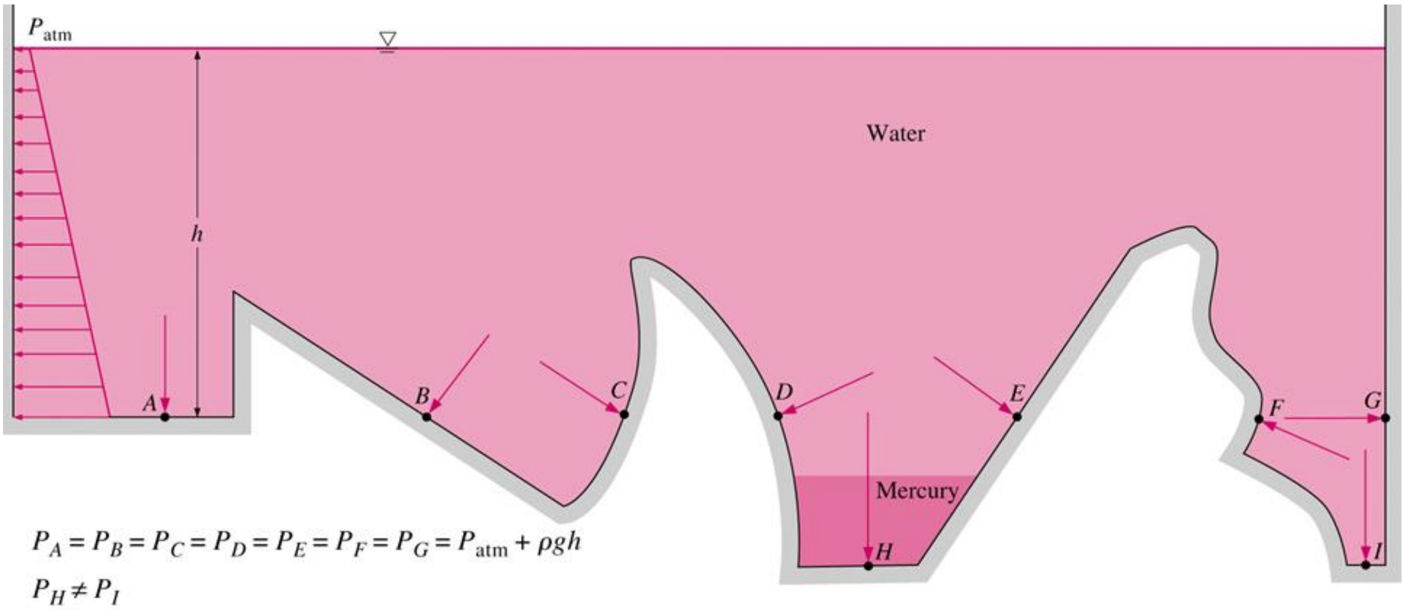
In the presence of a gravitational field, pressure increases with depth Fig. a, because more fluid rests on deeper layers. To obtain a relation for the variation of pressure with depth, consider rectangular element, Fig. b, and Force balance in *z*-direction gives

$\sum F_z = ma_z = 0 \quad \Rightarrow \quad p_2\Delta x - p_1\Delta x - \rho g \Delta x \Delta z = 0,$ Dividing by Δx and rearranging gives

$$\Delta p = p_2 - p_1 = \rho g \Delta z$$

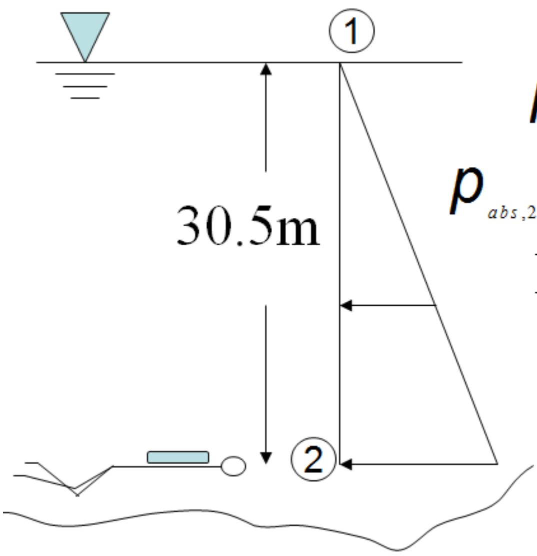


Pressure in a fluid at rest is independent of the shape of the container. And is the same at all points on a horizontal plane in a given fluid.



<u>Example:</u> Find the pressure on the diver at 30.5m deep under water and what is the danger of diver emergency ascent?

<u>Solution:</u>



Pressure on diver الغواص at 30.5m?

$$p_{gage,2} = \rho g z = \left(998 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)(30.5\text{m}) = 298.5\,kPa$$

$$p_{abs,2} = p_{gage,2} + p_{atm} = 298.5\,kPa + 101.325\,kPa = 399.825\,kPa$$

Danger of emergency ascent صعود?

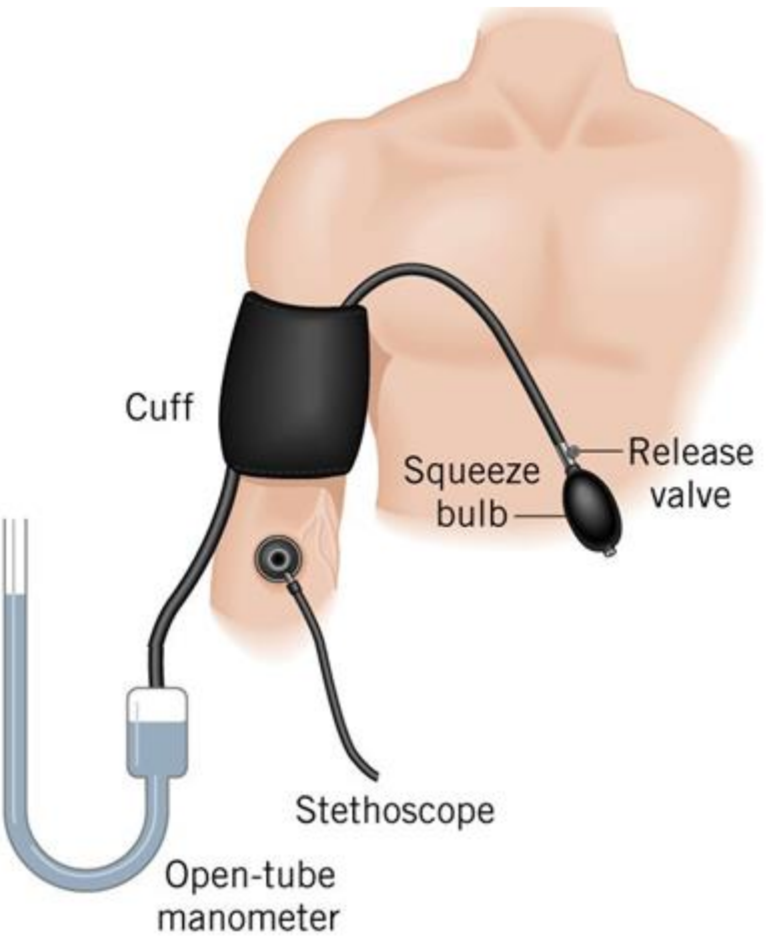
$$p_1 V_1 = p_2 V_2 \quad \text{Boyle's law (Temp.=const.)}$$

$$\frac{V_1}{V_2} = \frac{p_2}{p_1} = \frac{399.825\,kPa}{101.325\,kPa} \approx 4$$

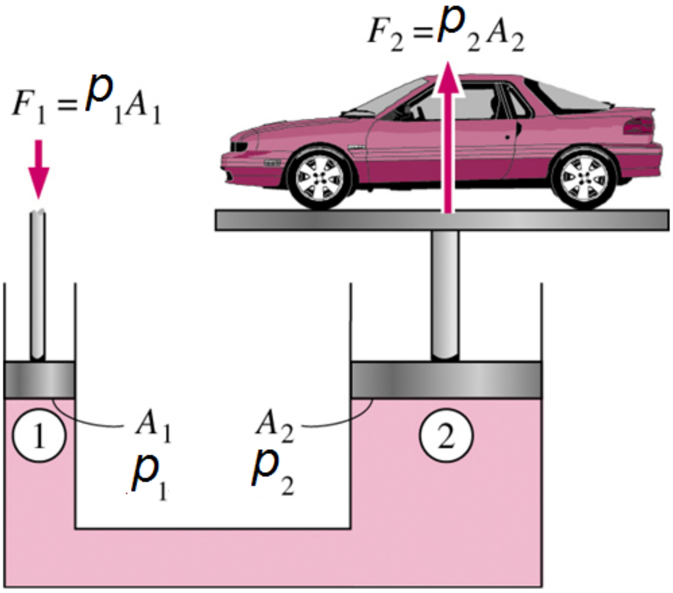
If you hold حبست your breath انفاسك on ascent, your lung رئتيك volume would increase by a factor of 4, which would result in embolism الاختناق and/or death الوفاة.

<u>Blood Pressure</u>

The blood pressure in your feet can be greater than the blood pressure in your head depending on whether a person is standing or reclining مستلقي .



Pascal's Law



Pressure applied to a confined محبوسfluid increases the pressure throughout by the same amount.

In picture, pistons are at same height:

$$p_1 = p_2 \rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2} \rightarrow \frac{F_2}{F_1} = \frac{A_2}{A_1}$$

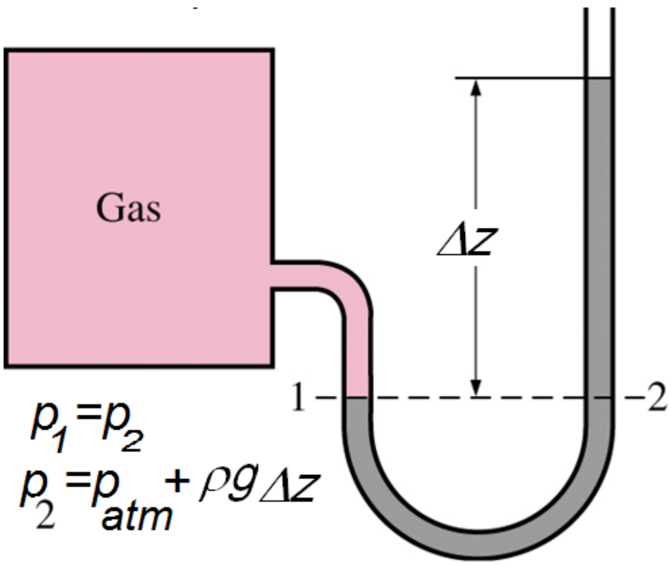
Ratio A_2/A_1 is called *ideal mechanical advantage*

Example: A hydraulic lift is to be used to lift a 2500 kg weight by putting a weight of 25 kg on a piston with a diameter of 10 cm. Determine the diameter of the piston on which the weight is to be placed.

Solution: $p_1 = p_2 \;, \; {}^{m_1 g}/_{A_1} = {}^{m_2 g}/_{A_2} \;\; \rightarrow \; D_2 = \sqrt{\left({}^{m_2}/_{m_1}\right) * D_1^2} = \sqrt{\left({}^{2500}/_{25}\right) * 10^2}$

$$D_2 = 100 \; cm$$

The Manometer



An elevation change of Δz in a fluid at rest corresponds to $\Delta p/\rho g$. A device based on this is called a **manometer.**

A manometer consists of a U-tube containing one or more fluids such as mercury, water, alcohol, or oil. Heavy fluids such as mercury are used if large pressure differences are anticipated يتوقع استخدامه.

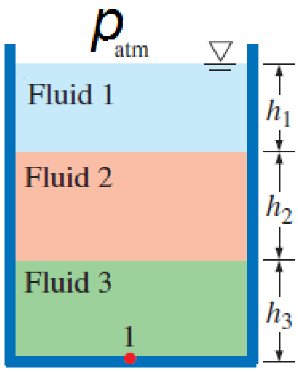
Example: Determine the static pressure difference indicated by an 18 cm column of fluid (liquid) with a specific gravity of 0.85.

$$\Delta P = \rho g h = S. \rho_{ref} gh = 0.85 * 1000 \; kg/m^3 * 9.81 m/s^2 * 0.18 \; m = 1501 \; N/m^2 \approx 1.5 \; kPa$$

For multi-fluid systems

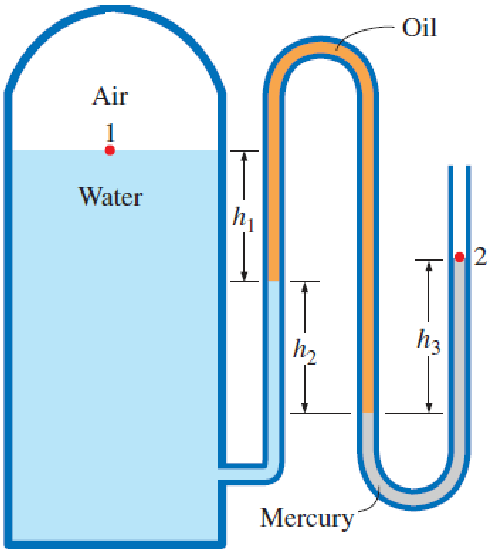
Pressure change across a fluid column of height h is $\Delta p = \rho g h$.

$$p_{atm} + \rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3 = p_1$$



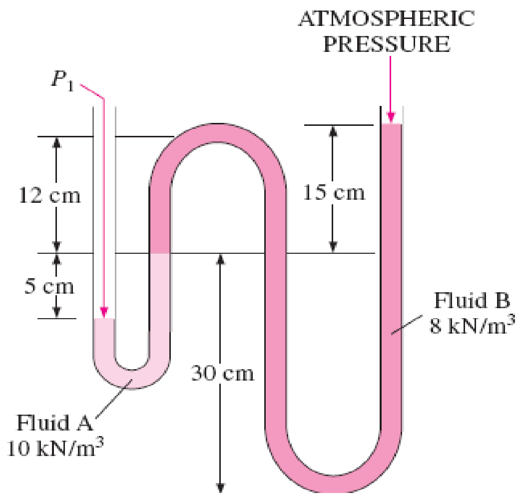
Mutlifluid Manometer

- Pressure increases downward, and decreases upward.
- Two points at the same elevation in a continuous fluid are at the same pressure.
- Pressure can be determined by adding and subtracting $\rho g h$ terms.



$$p_1 + \rho_{water} g h_1 + \rho_{oil} g h_2 - \rho_{mercury} g h_3 = p_2 = p_{atm}$$

Example: The pressure indicated by a manometer is to be determined. When the p_{atm}=758mmHg and the specific weights of fluid A and fluid B are given to be 10 kN/m^3 and 8 kN/m^3, respectively.



Analysis: The absolute pressure p_1 is determined from

$$p_1 = p_{atm} + (\rho g h)_A + (\rho g h)_B$$

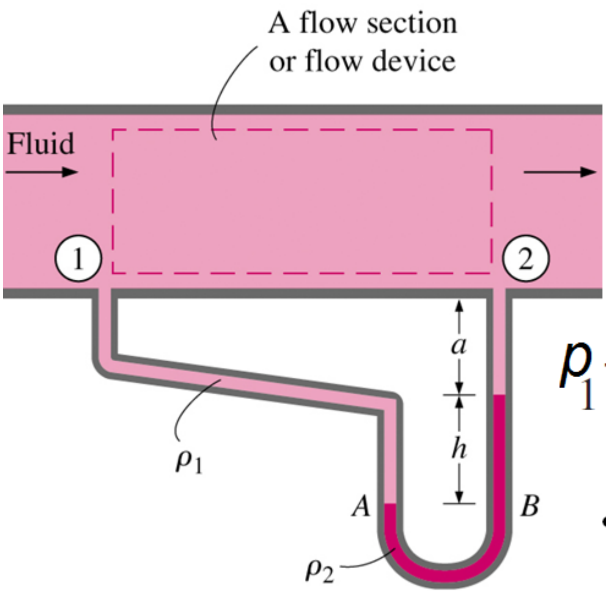
$$= (758 \text{ mm Hg})\left(\frac{0.1333 \text{ kPa}}{1 \text{ mm Hg}}\right)$$

$$+ (10 \text{ kN/m}^3)(0.05 \text{ m}) + (8 \text{ kN/m}^3)(0.15 \text{ m})$$

$$= \mathbf{102.7 \ kPa}$$

Note that 1 kPa = 1 kN/m^2.

Measuring Pressure Drops



- Manometers are well-suited to measure pressure drops across valves, pipes, heat exchangers, etc.
- Relation for pressure drop p_1-p_2 is obtained by starting at point 1 and adding or subtracting ρgh terms until we reach point 2.

$$P_1 + \rho_1 g(a+h) - \rho_2 gh - \rho_1 ga = P_2$$

$$P_1 - P_2 = (\rho_2 - \rho_1)gh$$

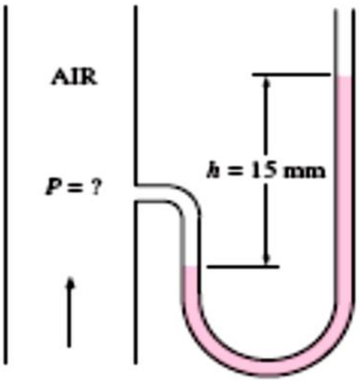
- If fluid in pipe is a gas, $\rho_2 >> \rho_1$ and P_1-$P_2 = \rho gh$

<u>Example</u>: The air pressure in a duct is measured by a mercury (ρ_{Hg} = 13,600 kg/m^3) manometer. For a given mercury-level difference between the two columns, the absolute pressure in the duct is to be determined. p_{atm}=100 kPa

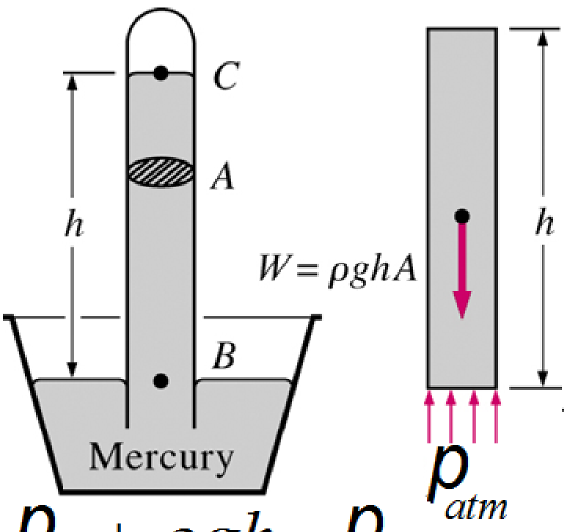
Analysis (*a*) The pressure in the duct is above atmospheric pressure since the fluid column on the duct side is at a lower level.

(*b*) The absolute pressure in the duct is determined from

$$p = p_{atm} + \rho gh = 100 + 13600 * 9.81 * 0.015 * 10^{-3} = 102\ kPa$$



<u>The Barometer</u>

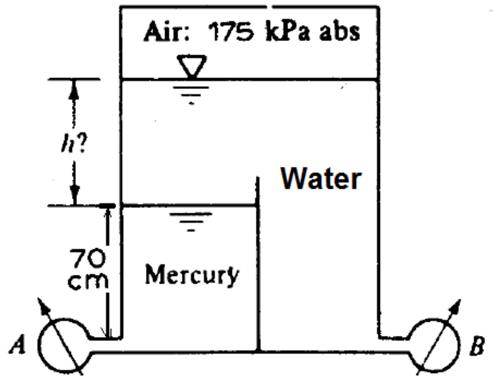


$$P_C + \rho gh = P_{atm}$$

$$P_{atm} = \rho gh$$

- Atmospheric pressure is measured by a device called a **barometer**; thus, atmospheric pressure is often referred to as the *barometric pressure*.
- p_C can be taken to be zero since there is only Hg vapor above point C, and it is very low relative to p_{atm}.
- Change in atmospheric pressure due to elevation has many effects: Cooking, nose bleeds, engine performance, aircraft performance.

Example: At 20 °C, gage *A* in Fig. reads 290 kPa abs. What is the height *h* of water? What does gage *B* read?



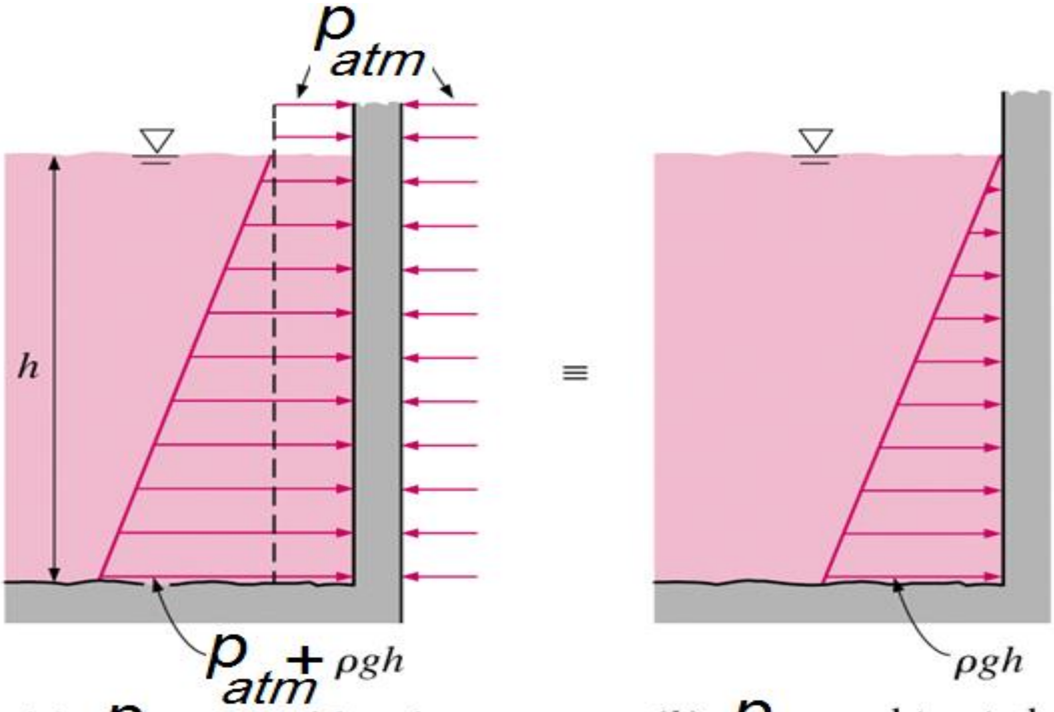
Solution: $290 - [(13.6)(9.81)](0.7) - 9.81h = 175$ $h = 2.227\ \text{m}$

$p_B - (9.81)(0.7 + 2.227) = 175$ $p_B = 204\ \text{kPa}$

Fluid Statics

- **Fluid Statics** deals with problems associated with fluids at rest.
- In fluid statics, there is no relative motion between adjacent fluid layers.
- Therefore, there is no shear stress in the fluid trying to deform it.
- The only stress in fluid statics is *normal stress*
  - o Normal stress is due to pressure
  - o Variation of pressure is due only to the weight of the fluid → fluid statics is only relevant in presence of gravity fields.
- Applications: Floating or submerged bodies, water dams and gates, liquid storage tanks, etc.

Hydrostatic Forces on Plane Surfaces

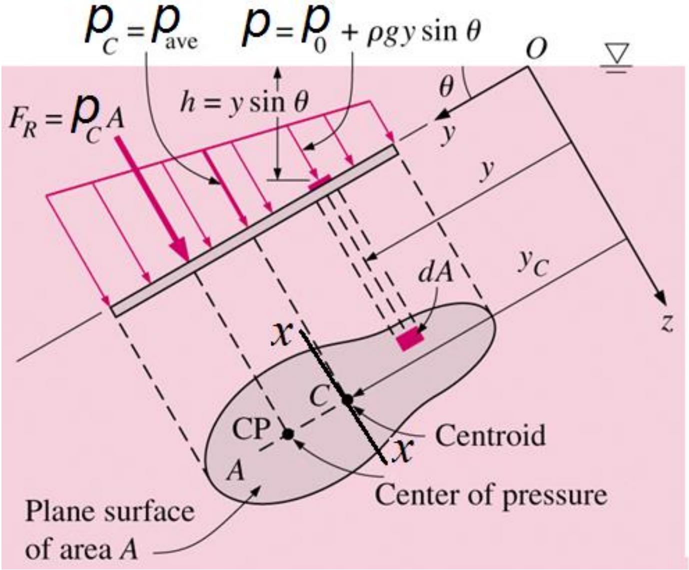


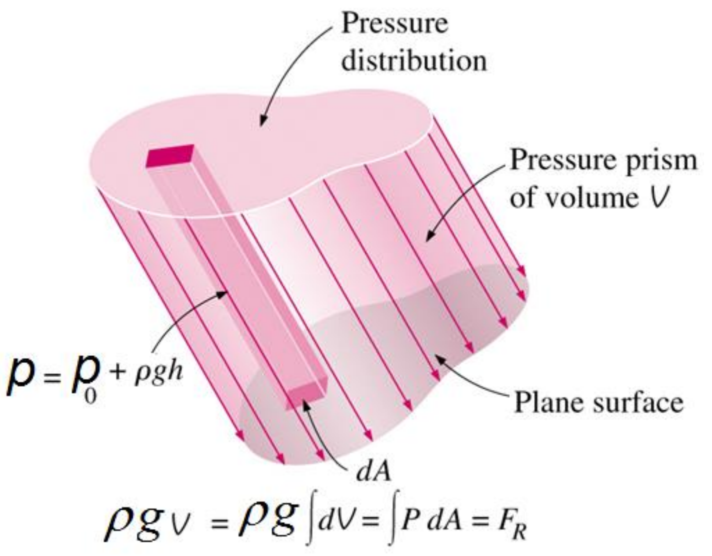
(*a*) p_{atm} considered   (*b*) p_{atm} subtracted

- On a *plane* surface, the hydrostatic forces form a system of parallel forces
- For many applications, magnitude and location of application, which is called **center of pressure**, must be determined.
- Atmospheric pressure p_{atm} can be neglected when it acts on both sides of the surface.

Resultant Force

The magnitude of Resultant Force F_R acting on a plane surface of a completely submerged plate in a homogenous fluid is equal to the product of the pressure p_C at the centroid of the surface and the area A of the surface



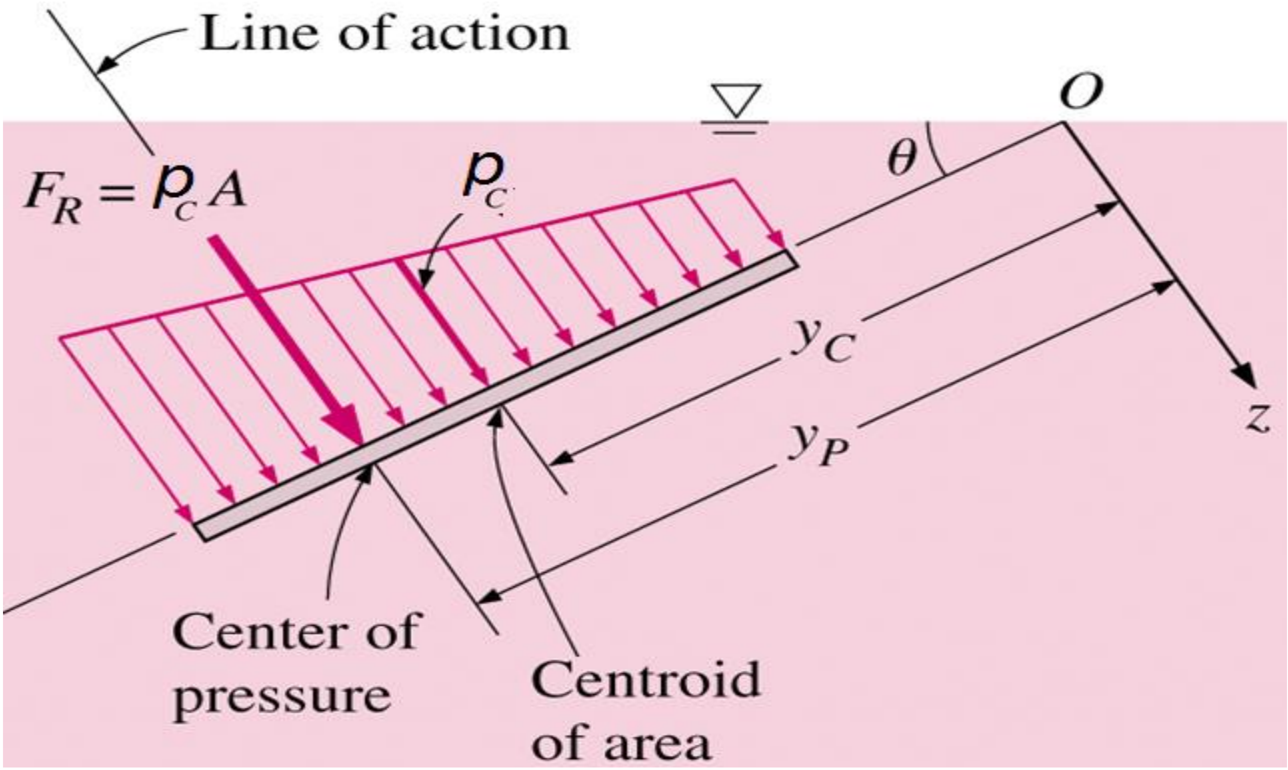


Centre of Pressure

- Line of action of resultant force $F_R = p_C A$ does not pass through the centroid of the surface. In general, it lies underneath where the pressure is higher.
- Vertical location y_p of **Center of Pressure** is determined by equation the moment of the resultant force to the moment of the distributed pressure force.

$$y_p = y_C + \frac{I_{xx,C}}{y_C A}$$

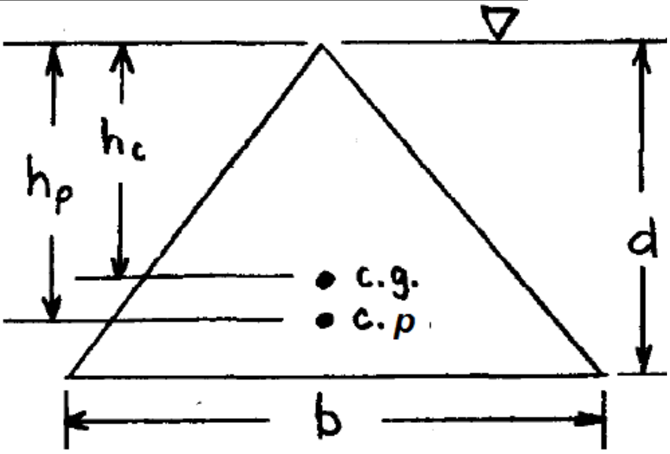
where $I_{xx,C}$ is the second moment of area of body at the centre of it about the x-x axis and is tabulated for simple geometries.



Example: If a triangle of height d and base b is vertical and submerged in liquid with its vertex at the liquid surface (see Fig.), derive an expression for the depth to its center of pressure.

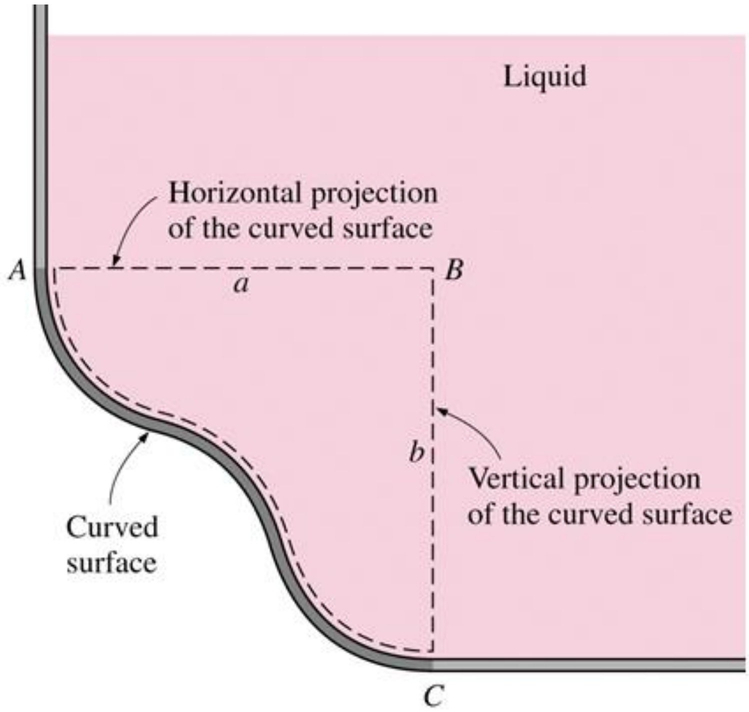
Solution: $h = y\sin\theta = y\sin 90 = y$, $I_{cg} = bd^3/36$

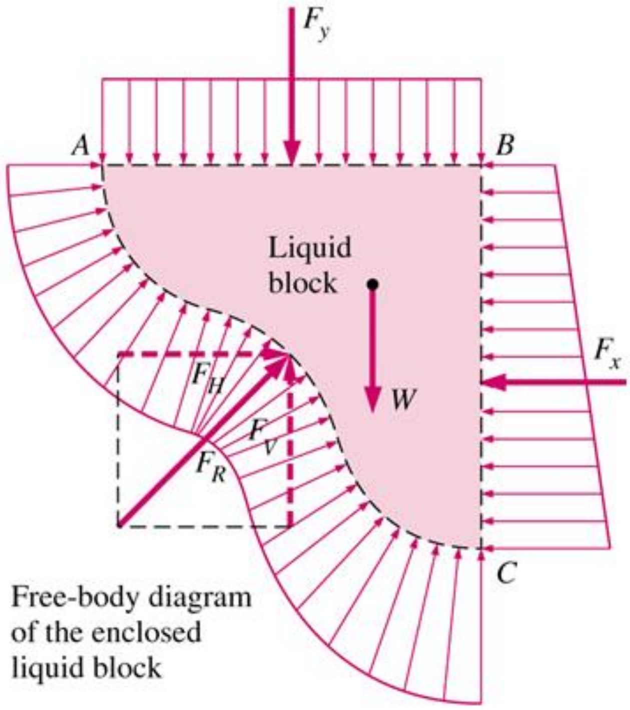
$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = \frac{2d}{3} + \frac{bd^3/36}{(2d/3)(bd/2)} = \frac{3d}{4}$$



Hydrostatic Forces on Curved Surfaces

- F_R on a curved surface is more involved since it requires integration of the pressure forces that change direction along the surface.
- Easiest approach: determine horizontal and vertical components F_H and F_V separately.
- Horizontal force component on curved surface: $F_H = F_x$. Line of action on vertical plane gives y coordinate of center of pressure on curved surface.
- Vertical force component on curved surface: $F_V = F_y + W$, where W is the weight of the liquid in the enclosed block $W = \rho g V$. x coordinate of the center of pressure is a combination of line of action on horizontal plane (centroid of area) and line of action through volume (centroid of volume).
- Magnitude of force $F_R = (F_H^2 + F_V^2)^{1/2}$. and angle of force is $\alpha = \tan^{-1}(F_V/F_H)$.





Example: Compute the horizontal and vertical components of the hydrostatic force on the quarter-circle face of the tank shown in Fig. *a*.

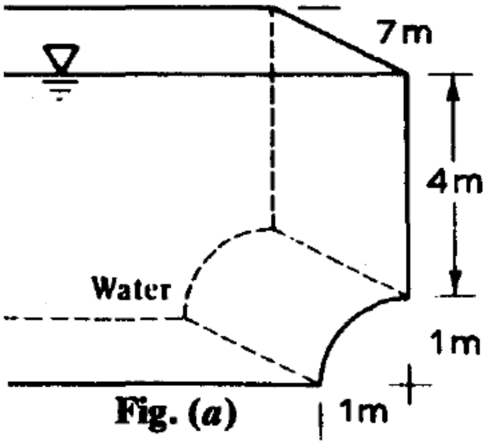


Fig. (*a*)

Solution: $F_H = \rho g h_{cg} A = 9.81[4+\frac{1}{2}][(1)(7)] = 308\ \text{kN}$

$F_V = F_1 - F_2$ (See Fig. *b*.)

$= (9.81)[(7)(1)(5)] - (9.81)[(7)(\pi)(1)^2/4] = 289\ \text{kN}$

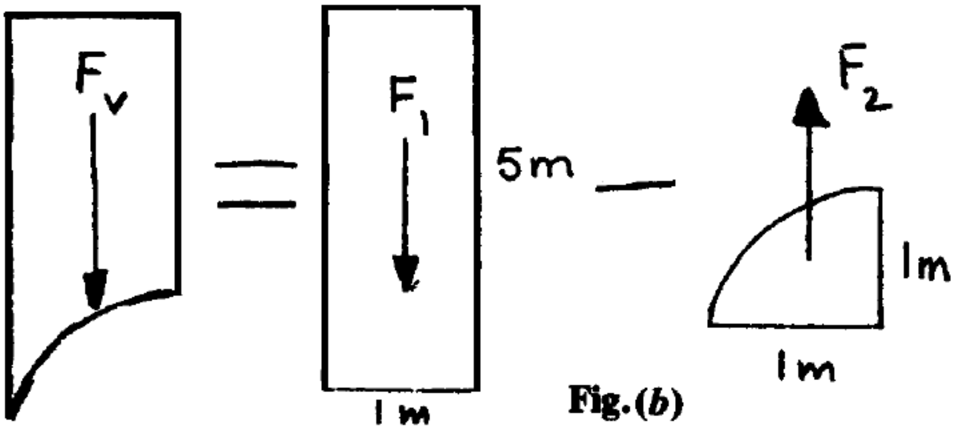
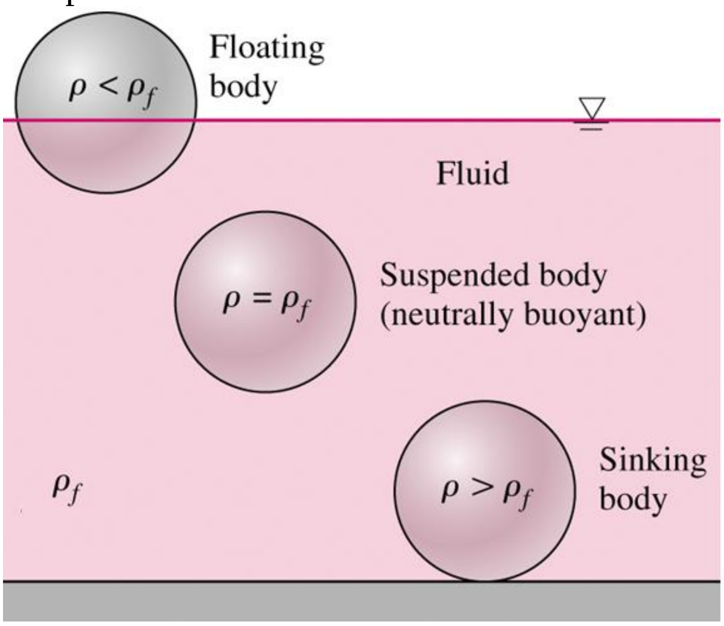


Fig. (*b*)

Buoyancy and Stability

- Buoyancy is due to the fluid displaced by a body. $F_B = \rho_f gV$.
- **Archimedes principal** : The buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body, and it acts upward through the centroid of the displaced volume.



- Buoyancy force F_B is equal only to the displaced volume $\rho_f g V_{displaced}$.
- Three scenarios possible
  1. $\rho_{body} < \rho_{fluid}$ Floating body
  2. $\rho_{body} = \rho_{fluid}$ Neutrally buoyant
  3. $\rho_{body} > \rho_{fluid}$ Sinking body

Example A stone weighs 105 N in air. When submerged in water, it weighs 67.0 N. Find the volume and specific gravity of the stone.

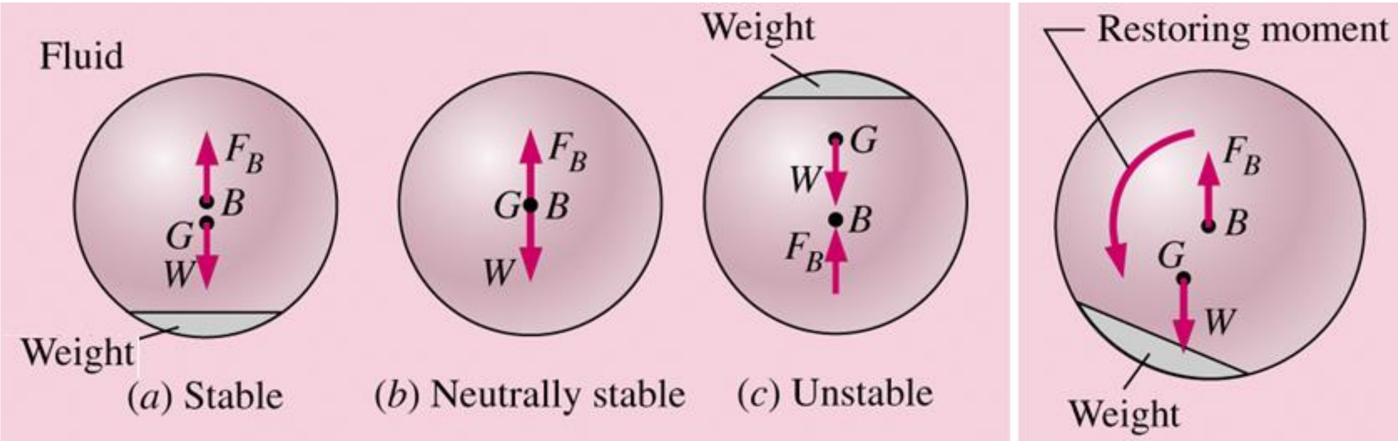
Solution: Buoyant force (F_b) = weight of water displaced by stone (W)

$W = 105 - 67.0 = 38.0\ \text{N}$ $W = \rho g V = 9810\ V$ $38.0 = 9810V$ $V = 0.00387\ \text{m}^3$

$$S = \frac{\text{weight of stone in air}}{\text{weight of equal volume of water}} = \frac{105}{38.0} = 2.76$$

Stability of Immersed Bodies

- Rotational stability of immersed bodies depends upon relative location of *center of gravity G* and *center of buoyancy B.*
  - *G* below *B*: stable
  - *G* above *B*: unstable
  - *G* coincides with *B*: neutrally stable.



Example: Determine the magnitude and direction of the force necessary to hold a concrete cube, 0.300 m on each side, in equilibrium and completely submerged **(*a*)** in mercury (Hg) and **(*b*)** in water. Use $S_{concrete}=2.40$.

Solution: **(*a*)**Since $S_{Hg}=13.6$ and $S_{concrete}=2.40$, it is evident that the concrete will float in mercury. Therefore, a force F acting downward will be required to hold the concrete in equilibrium and completely submerged in mercury. The forces acting on the concrete are shown in Fig.*a*, where F is the force required to hold the concrete cube in equilibrium and completely submerged, W is the weight of the concrete cube in air, and F_b is the buoyant force. $\sum F_y = 0$, $F + W - F_b = 0$, $F + [(2.40)(9.81)][(0.300)(0.300)(0.300)] - [(13.6)(9.81)][(0.300)(0.300)(0.300)] = 0$, $F = 2.96$ kN (downward). **(*b*)** Since $S_{concrete} = 2.40$, it will sink in water. Therefore, a force F acting upward will be required to hold the concrete in equilibrium and completely submerged in water. The forces acting on the concrete in this case are shown in Fig.*b*. $\sum F_y = 0$, $W-F-F_b=0$, $[(2.40)(9.81)][(0.300)(0.300)(0.300)] - F - 9.81[(0.300)(0.300)(0.300)] = 0$, $F = 0.370$ kN (upward).

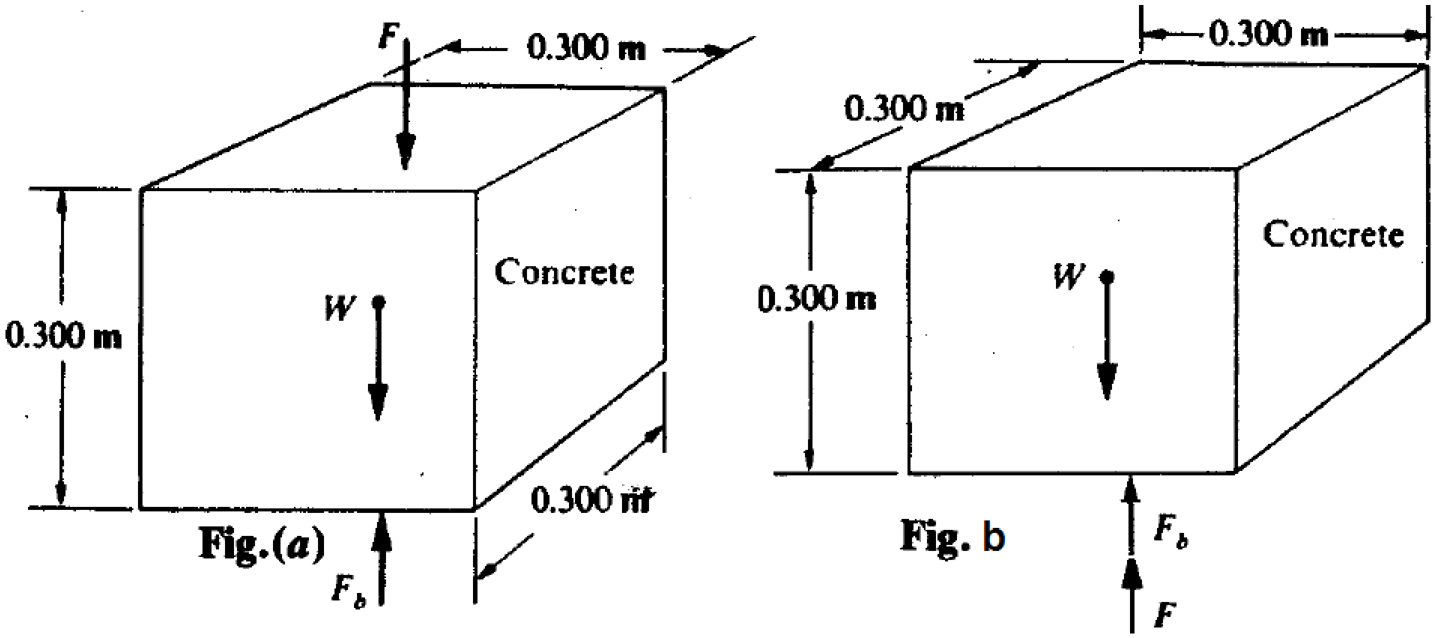
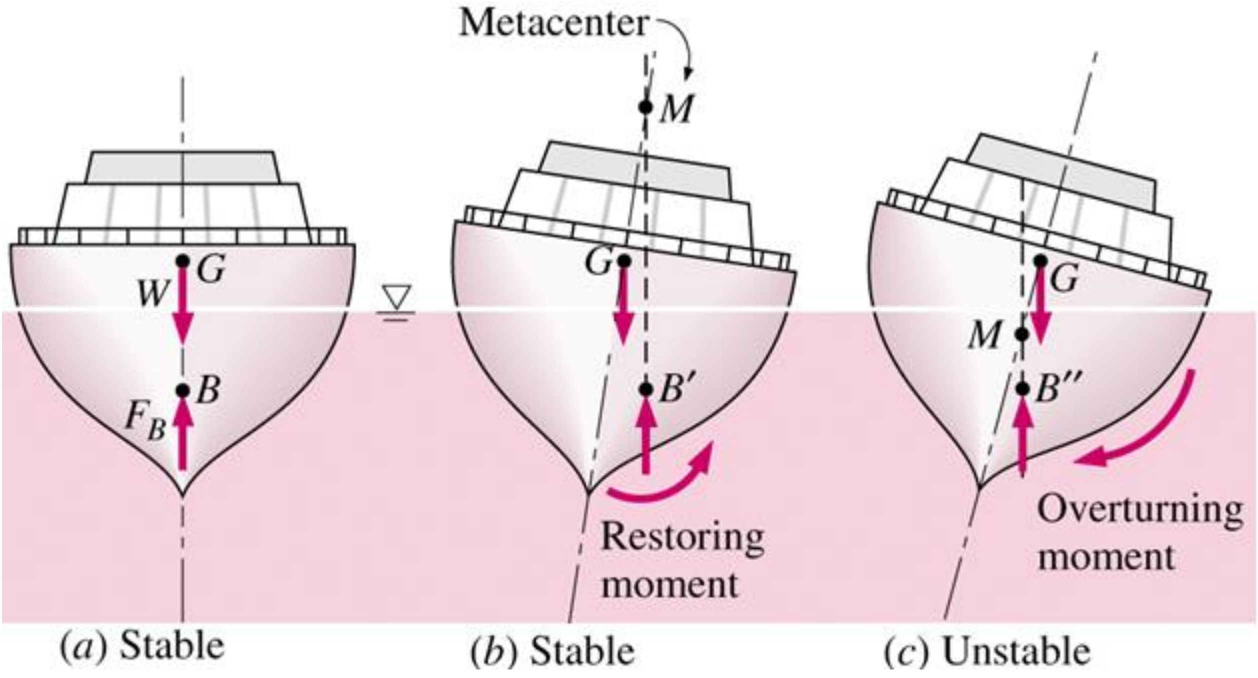


Fig.(*a*) Fig. b

Stability of Floating Bodies

- If body is bottom heavy (*G* lower than *B*), it is always stable.
- Floating bodies can be stable when *G* is higher than *B* due to shift in location of center buoyancy and creation of restoring moment.

- Measure of stability is the metacentric height G*M*. If *GM*≥0, ship is stable.



Rigid-Body Motion

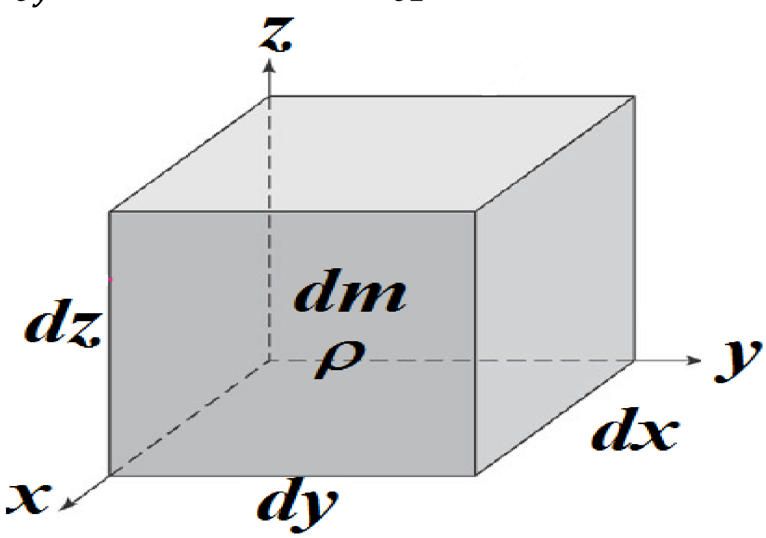
There are special cases where a body of fluid can undergo rigid-body motion: linear acceleration, and rotation of a cylindrical container.

In these cases, no shear is developed. Assume accelerating of a fluid element *of mass dm* and volume *dxdydz*. Newton's 2nd law of motion can be used to derive an **equation of motion** for this element as (Force= mass*acceleration) or (F=m*a) and in differential form ($d\vec{F} = -dm * \vec{a}$) or ($d\vec{F} = -\rho dxdydz * \vec{a}$). But this equation can be writing in partial differential in Cartesian coordinate form: $\partial F_x = -\rho \partial x \partial y \partial z * a_x$, $\partial F_y = -\rho \partial x \partial y \partial z * a_y$, *and* $\partial F_z = -\rho \partial x \partial y \partial z * (a_z + g)$.

These equations can be re-arranged as

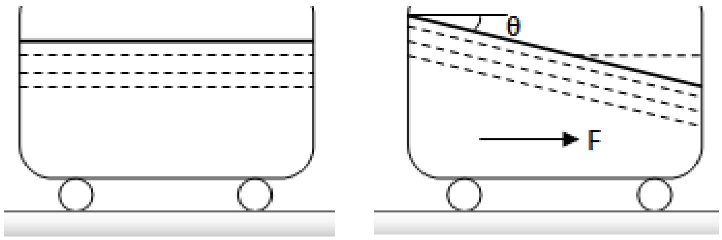
$$\frac{\partial F_x}{\partial x \partial y \partial z} = -\rho * a_x \; , \; \frac{\partial F_x}{\partial x \partial y \partial z} = -\rho * a_y \; , and \; \frac{\partial F_x}{\partial x \partial y \partial z} = -\rho * (a_z + g)$$

Or $$\boldsymbol{\frac{\partial p}{\partial x} = \rho * a_x \; , \; \frac{\partial p}{\partial y} = \rho * a_y \; , and \; \frac{\partial p}{\partial z} = \rho * (a_z + g)}$$

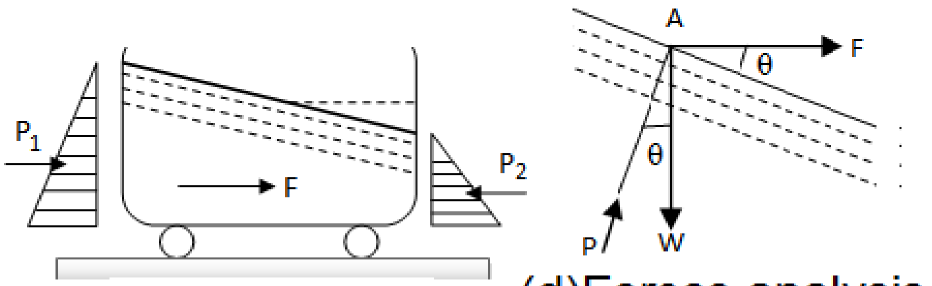


Linear Acceleration

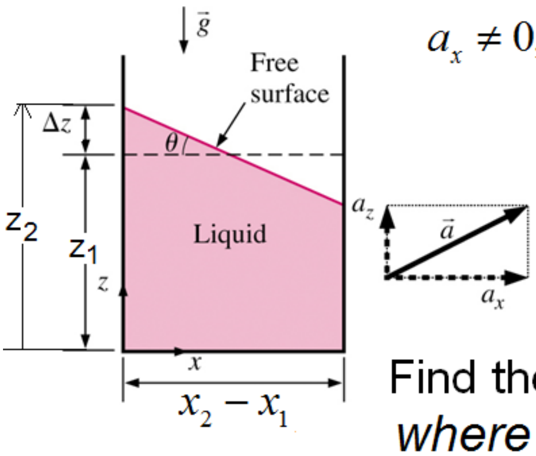
For a container moves on a straight path



(a) Before acceleration (b) After acceleration



(c) Pressure on sides (d)Forces analysis



$a_x \neq 0, a_y = a_z = 0$ $\quad \dfrac{\partial p}{\partial x} = \rho a_x, \dfrac{\partial p}{\partial y} = 0, \dfrac{\partial p}{\partial z} = -\rho g$

Total differential of p $dp = -\rho a_x dx - \rho g dz$

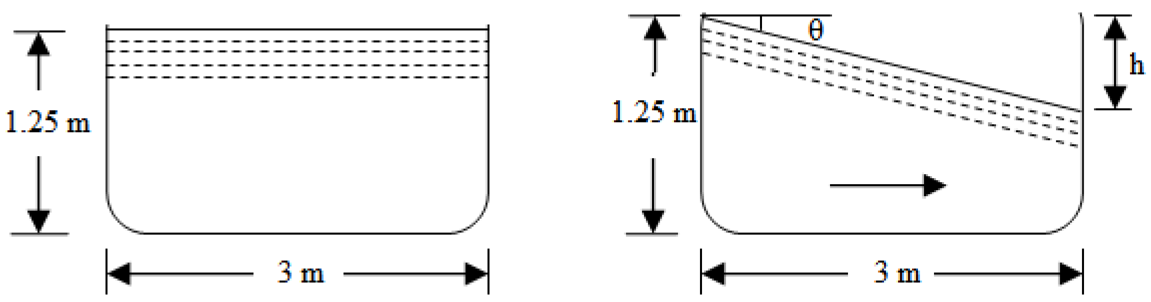
Pressure difference between 2 points

$$p_2 - p_1 = -\rho a_x (x_2 - x_1) - \rho g (z_2 - z_1)$$

Find the rise by selecting 2 points on free surface

where $p_2 = p_1$ *therefore* $\Delta z = z_2 - z_1 = -\dfrac{a_x}{g}(x_2 - x_1)$

Example: An open rectangular tank 3m long. 2.5m wide and 1.25m deep is completely filled with water. If the tank is moved with an acceleration of 1.5m/s^2, find the slope of the free surface of water and the quantity of water which will spill out of the tank.



(a) Before acceleration (b) After acceleration

Given, l = 3 m, b = 2.5 m, d = 1.25 m, a = 1.5 m/s^2

Slope of the free surface of water

θ = Angle which the free surface of water will make with the horizontal.

$$\tan\theta = \frac{a}{g} = \frac{1.5}{9.81} = 1.53 \quad \Rightarrow \theta = 8.7^\circ$$

Quantity of water which will spill out of the tank

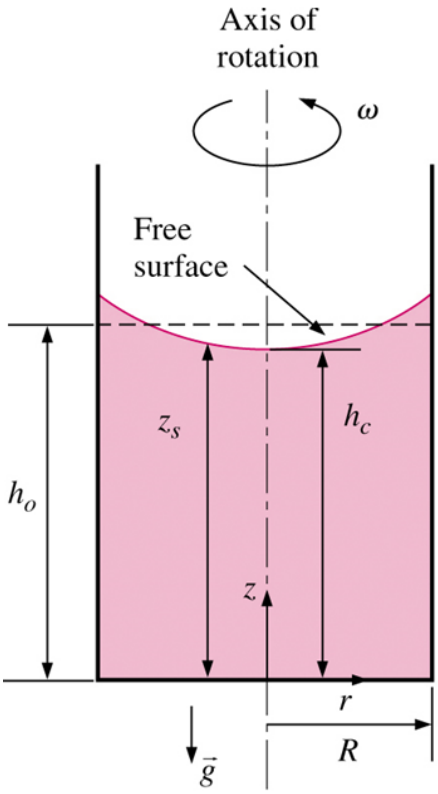
From the above figure we can see that the depth of water on the front side,

$$h = 3\tan\theta = 3 \times 0.153 = 0.459\ m$$

∴ Quantity of water which will spill out of the tank,

$$V = \frac{1}{2} \times 3 \times 2.5 \times 0.459 = 1.72\ m^3 = 1720\ litres$$

<u>Rotation in a Cylindrical Container</u>



Container is rotating about the z-axis

$$a_r = -r\omega^2 \quad , \quad a_\theta = a_z = 0$$

$$\frac{\partial p}{\partial r} = \rho r\omega^2, \quad \frac{\partial p}{\partial \theta} = 0, \quad \frac{\partial p}{\partial z} = -\rho g$$

Total differential of p

$$dp = \rho r\omega^2 dr - \rho g dz$$

On an isobar, dp = 0 Equation of the free surface

$$\frac{dz_{isobar}}{dr} = \frac{r\omega^2}{g} \longrightarrow z_{isobar} = \frac{\omega^2}{2g}r^2 + C_1$$

<u>Exercise</u>: complete the equation of the isobar, in terms of the original surface height, h_0, in the absence of rotation.

<u>Solution</u>: $z_{isobar} = \frac{\omega^2 r^2}{2g} + C_1$, $at\ r = 0$, $z_{isobar} = h_c \rightarrow C_1 = h_c$

Therefore $z_{isobar} = \frac{\omega^2 r^2}{2g} + h_c$

To evaluate the isobar, in terms of the original surface height, h_0, in the absence of rotation, we can write the volume of the liquid $V = \pi R^2 h_o$ and

$$\pi R^2 h_o = 2\pi \int_0^R \int_0^z r dr dz = 2\pi \int_0^R r dr [z]_0^z = 2\pi \int_0^R r dr z$$

Substitute the value of z_{isobar} in z in the Eq. above we have

$$R^2 h_o = 2\int_0^R rdr(\frac{\omega^2 r^2}{2g} + h_c) = \int_0^R \frac{\omega^2 r^3}{g} dr + 2h_c \int_0^R rdr$$

$$R^2 h_o = \frac{\omega^2 R^4}{4g} + 2h_c \frac{R^2}{2} \quad \rightarrow \quad h_o = \frac{\omega^2 R^2}{4g} + h_c$$

But $h_c = z_{isobar} - \frac{\omega^2 r^2}{2g}$ therefore $h_o = \frac{\omega^2 R^2}{4g} + z_{isobar} - \frac{\omega^2 r^2}{2g}$ and

$$z_{isobar} = h_o - \frac{\omega^2}{4g}(R^2 - 2r^2)$$

Example: An open cylindrical container 0.5m in diameter and 0.8m in height ,filled with oil up-to 0.5 m and rotating about its vertical axis. a) Find the speed at which the liquid will start to spill over ينسكب, b) the speed at which the point of the bottom centre will just exposed السرعة التي عندها تظهرللتو نقطة مركز قعرالاسطوانة, and c) how much oil will spill over in case b (Take the specific gravity of liquid is 0.88).

Solution: Given data: Diameter of the cylinder *R=0.5 m*, height of the cylinder 0.8 m, and height of the oil before cylindrical container rotates is $h_o = 0.5\ m$

a) The liquid will start to spill over when the maximum height at the periphery becomes 0.8 m at *r=R= 0.5m*.

$$z_{isobar} = h_o - \frac{\omega^2}{4g}(R^2 - 2r^2) \implies \omega^2 = (h_o - z_{isobar}) * \frac{4g}{(R^2 - 2r^2)}$$

$$\omega = \sqrt{(h_o - z_{isobar}) * \frac{4g}{(R^2 - 2r^2)}} = \sqrt{(0.5 - 0.8) * \frac{4(9.81)}{(0.5^2 - 2(0.5)^2)}} = 6.86\ rad/s$$

b) The bottom of the centre will expose when z_{isobar} will be zero at *r=0*. Therefore,

$$\omega = \sqrt{(h_o - z_{isobar}) * \frac{4g}{(R^2 - 2r^2)}} = \sqrt{(0.5 - 0) * \frac{4(9.81)}{(0.5^2 - 2(0)^2)}} = 8.86\ rad/s$$

c) Homework