INTRODUCTION TO ROBOTICS

Lecture 5

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This lecture is going to answer the question:

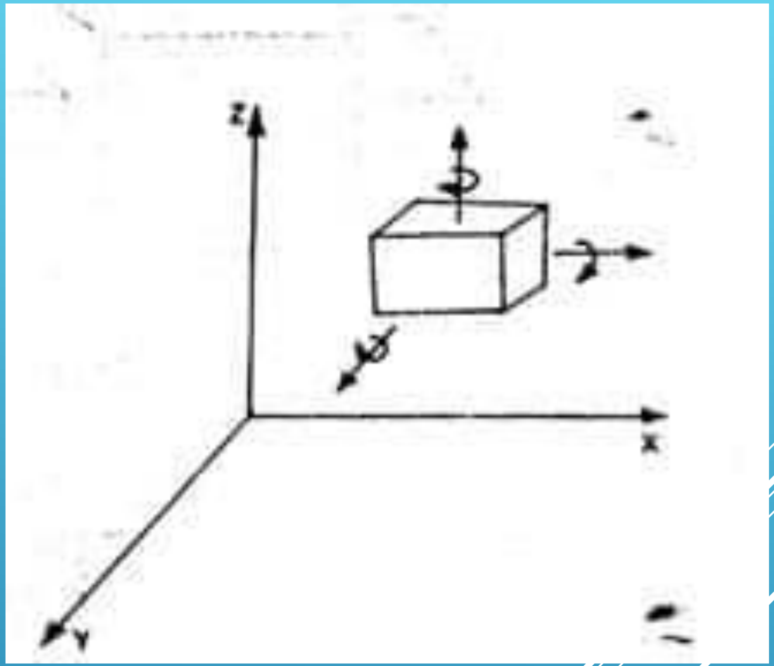
How many joints can be controller independently?

There are two versions of formula:

- 2D version
- 3D version

What is Degrees of freedom (DOF) :

- It is the number of independent coordinates required to describe the position of a body in space .
- A free body in space can have six degrees of freedom. i.e., linear positions along x, y and z axes and rotational/angular positions with respect to x, y and z axes.
- An object in free space has six degrees of Freedom. A fixed object has zero degree of freedom.



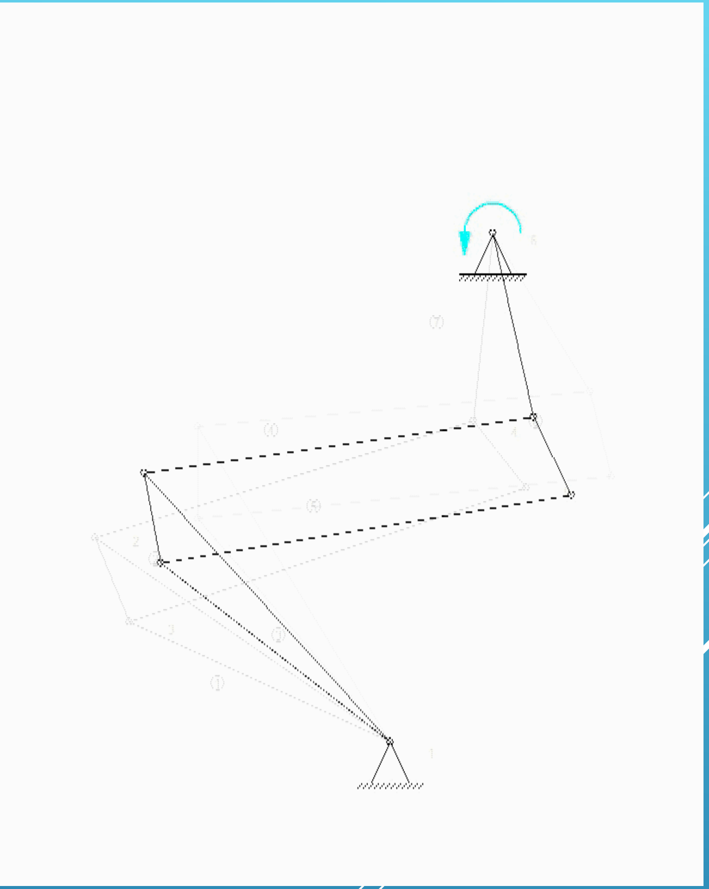
Gruebler's Equation

To design a mechanism, the first thing we should check is the number of degrees of freedom (DOF) of the mechanism, which is the inputs required to control the position of all links of the mechanism. It's usually the number of actuators needed to operate the mechanism.

when we require one degree of freedom it means we need to control the position all linkages with only 1 actuator.

have 1 degree of freedom which requires only 1 actuator to move the mechanism. It could be a motor, an air cylinder, etc.

Below we apply Grubler’s formula to several planar and spatial mechanisms. We distinguish between two types of mechanism: open-chain mechanisms (also known as serial mechanisms) and closed-chain mechanisms. A closed-chain mechanism is any mechanism that has a closed loop.



GRUBLER LAW FOR PLANAR ROBOTS

Proposition 2.1. *Consider a planar robot consisting of N links, where the ground is also regarded as a link. Let J denote the total number of joints, and let f_i be the degrees of freedom of the i-th joint. The degrees of freedom (dof) of the robotic mechanism can then be evaluated from the formula*

$$
\begin{aligned}
dof &= 3(N-1) - \sum_{i=1}^{J} (3 - f_i) \\
&= 3(N-1-J) + \sum_{i=1}^{J} f_i \qquad (2.10)
\end{aligned}
$$

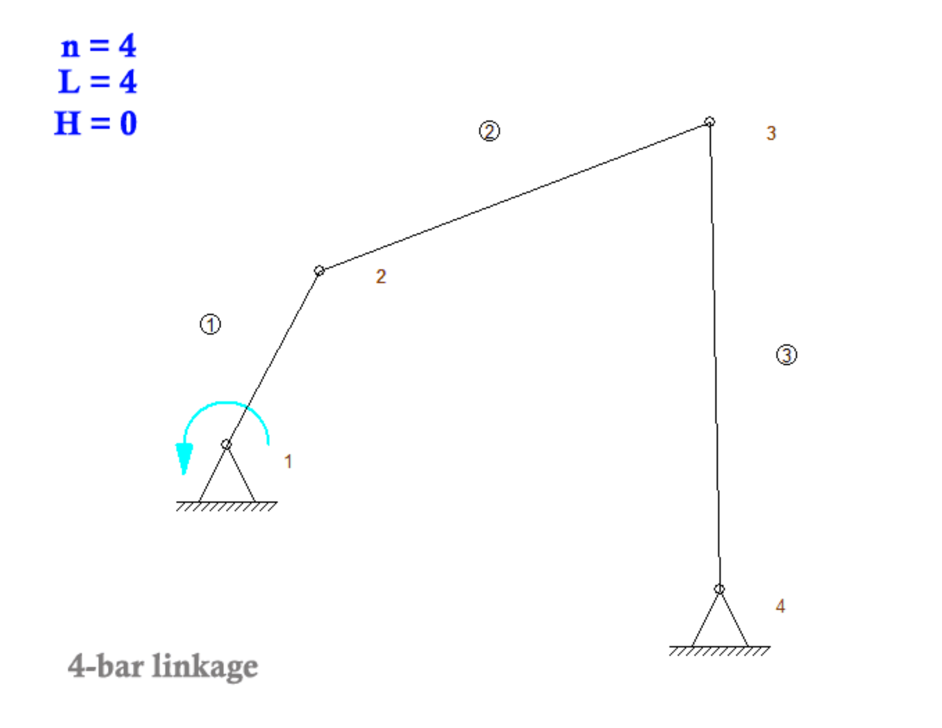
Grübler criterion determines the number of DOF of a linkages (mechanisms), that is, a coupling of rigid bodies by means of mechanical constraints (joints).

The **Grübler** criterion is also called the mobility formula, because it computes the number of parameters that define the configuration of a linkage from the number of links and joints and the degree of freedom at each joint.

GRUBLER LAW FOR PLANAR ROBOTS

The **four-bar linkage** shown has 4 links (3 bars with 1 ground link) and 4 revolute joints which the degree of freedom (F) can be calculated as follows.

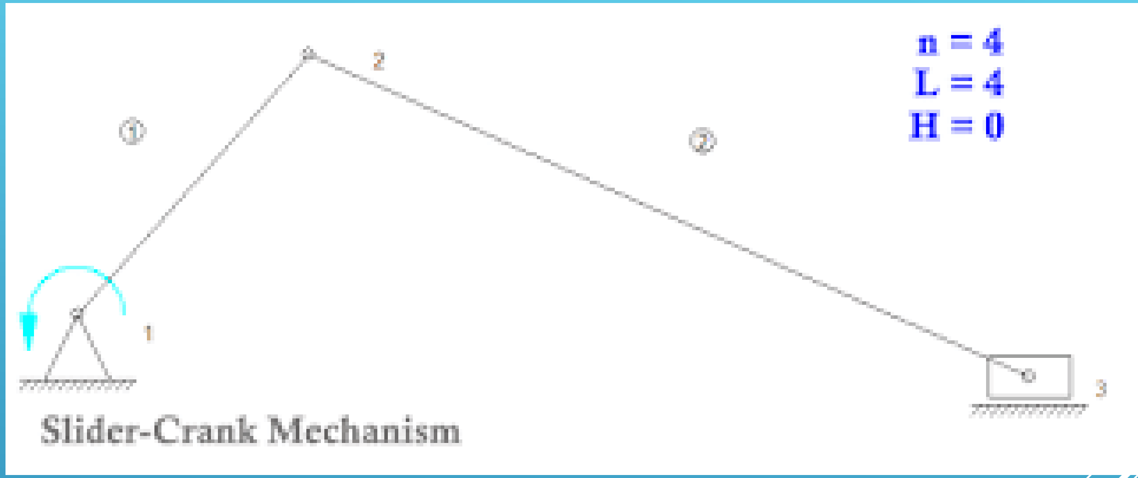
N = 4 --- 4 links
J = 4 --- 4 revolute joints
DOF = 3(4-4-1) +4*1= 1



GRUBLER LAW FOR PLANAR ROBOTS

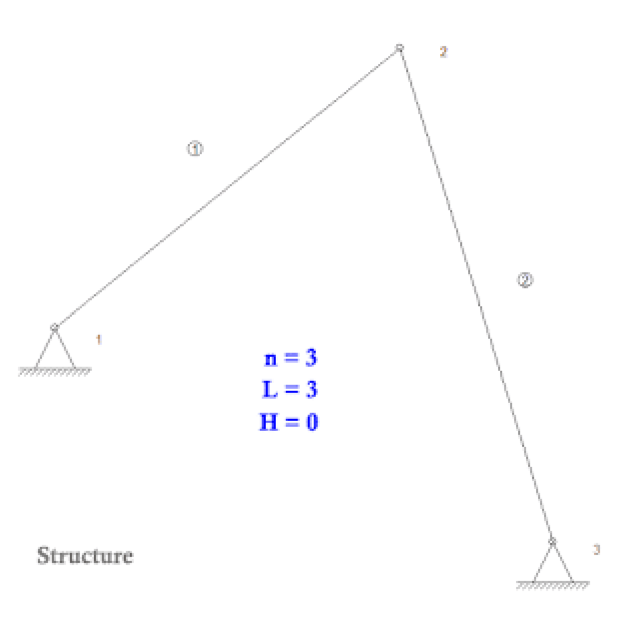
the **slider-crank mechanism** shown has the number of links and joints as follows:

N = 4 --- 2 links + 1 ground link + 1 slider
J = 4--- 3 pins + 1 slider
DOF = 3(4-4-1) + 4*1= 1



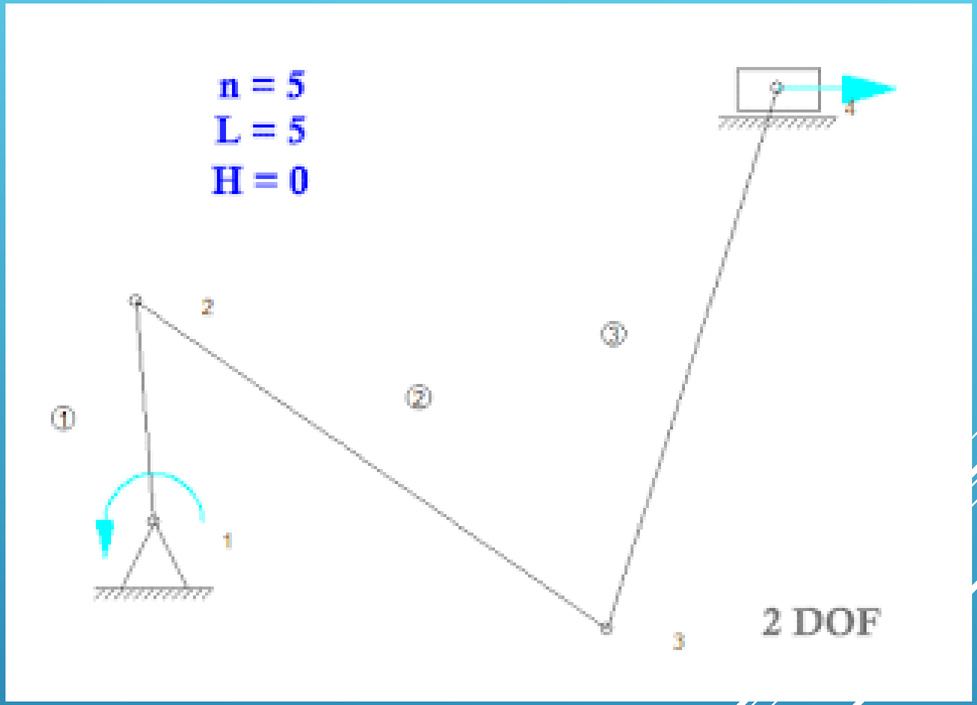
if we change the slider joint of the slider-crank mechanism to the fixed pin joint, the mechanism will be locked since it has 0 DOF which is considered as a structure. The calculation using Gruebler's equation is as follows

N = 3 --- 2 links + 1 ground link
J = 3 --- 3 pin joints
DOF = 3(3-3-1) + 3*1= 0 --- then it can't move.



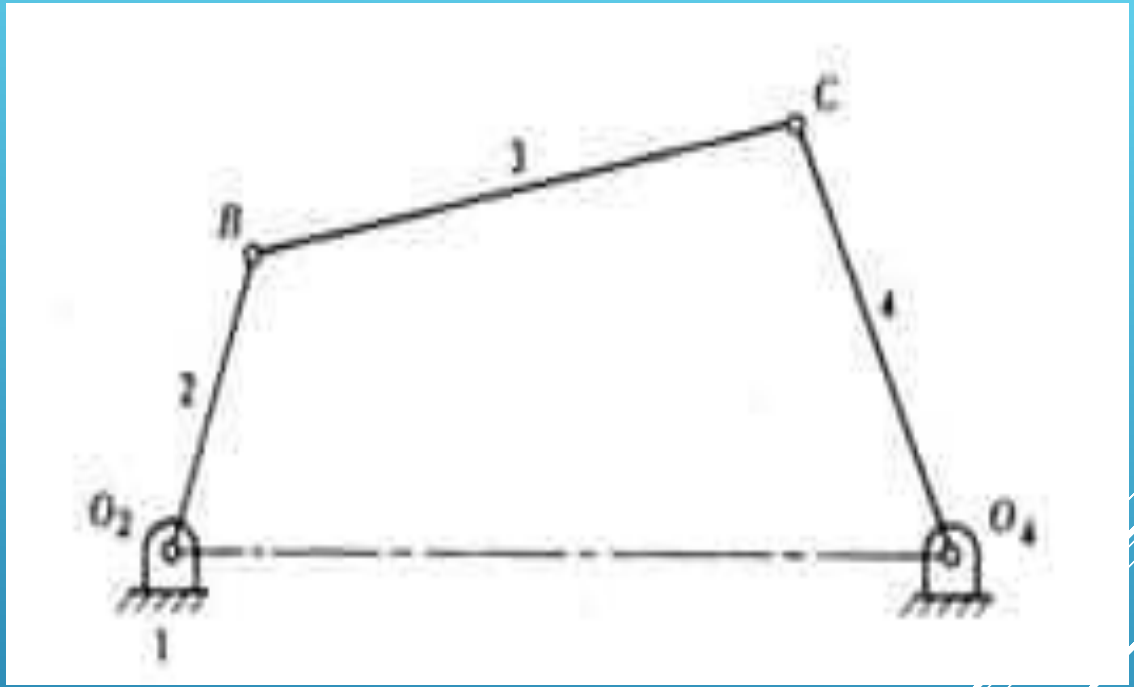
GRUBLER LAW FOR PLANAR ROBOTS

N = 5 --- 3 links + 1 slider + 1 ground link
J = 5 --- 4 pin joints + 1 slider joint
DOF = 3(5-5-1) + (5*1) = 2 --- F > 1, the mechanism is unconstrained.

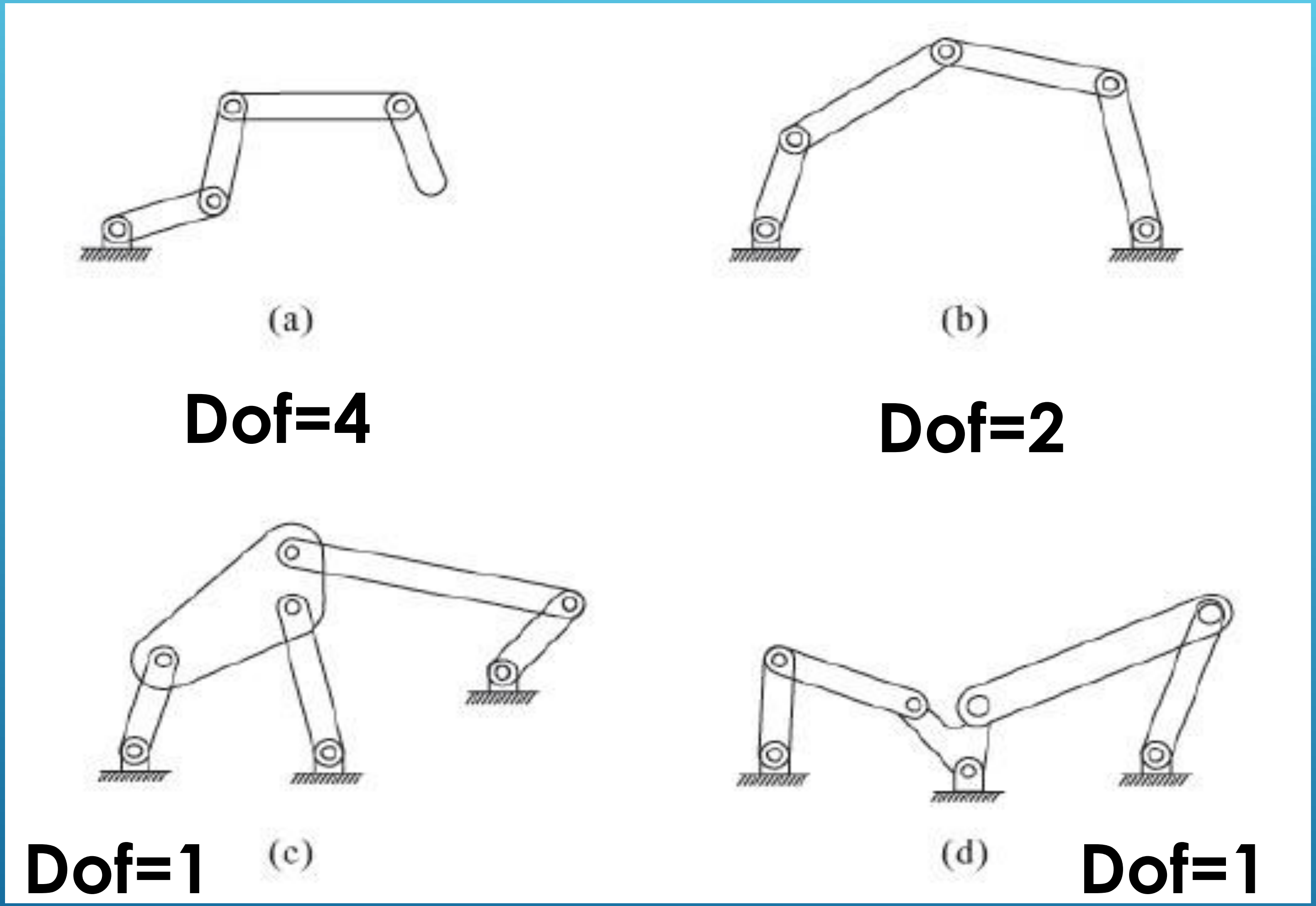


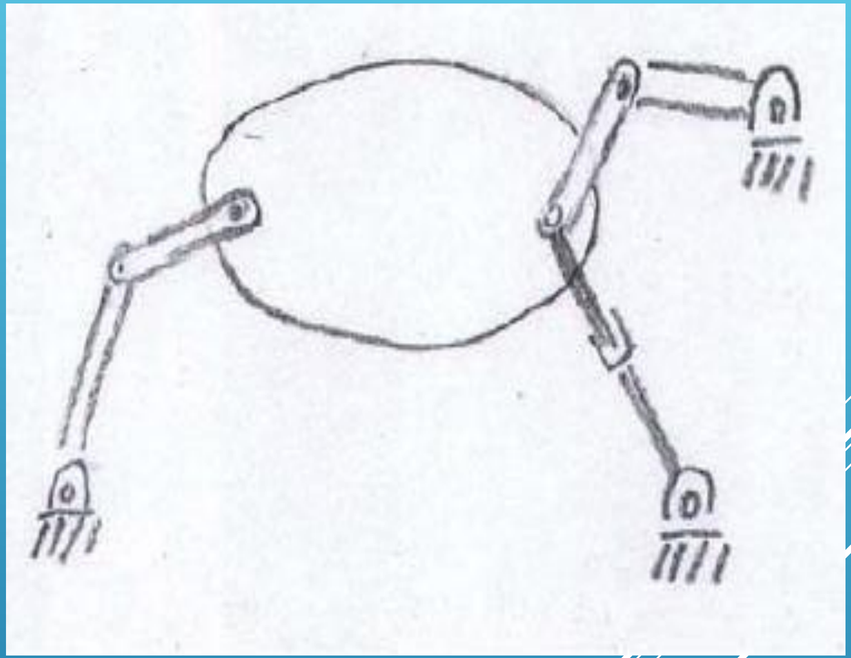
GRUBLER LAW FOR PLANAR ROBOTS

n = 4 --- 3links + 1 ground link
J = 4 --- 4 pins
DOF = 3(4-4-1) + 4*1= 1



GRUBLER LAW FOR PLANAR ROBOTS





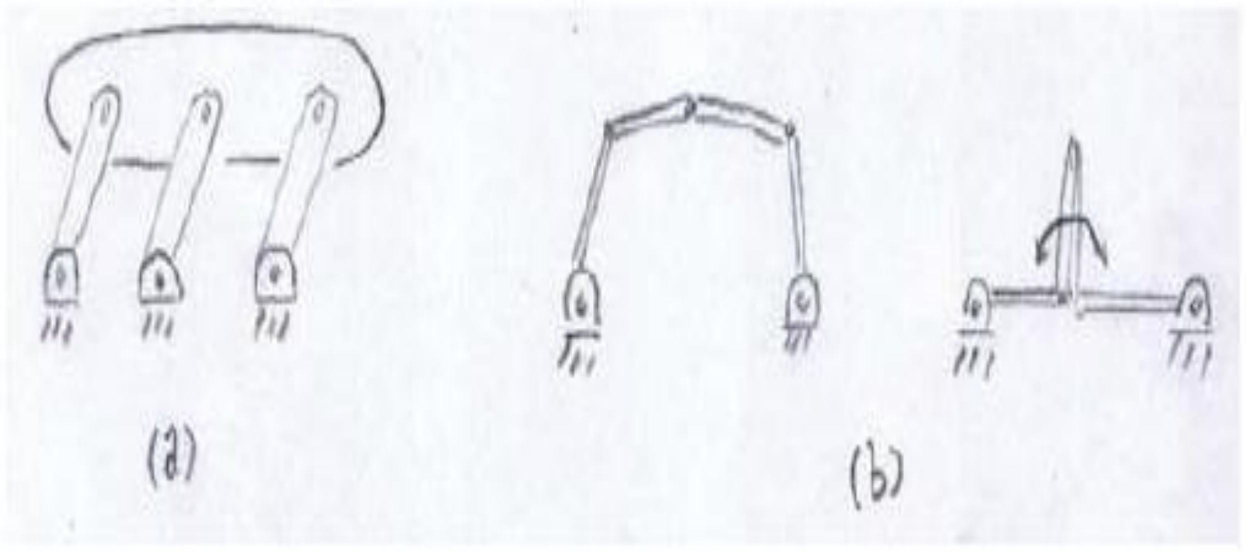
Dof=3

GRUBLER LAW FOR PLANAR ROBOTS

GRUBLER FORMULA IS NOT FEASIBLE ALL THE TIME!

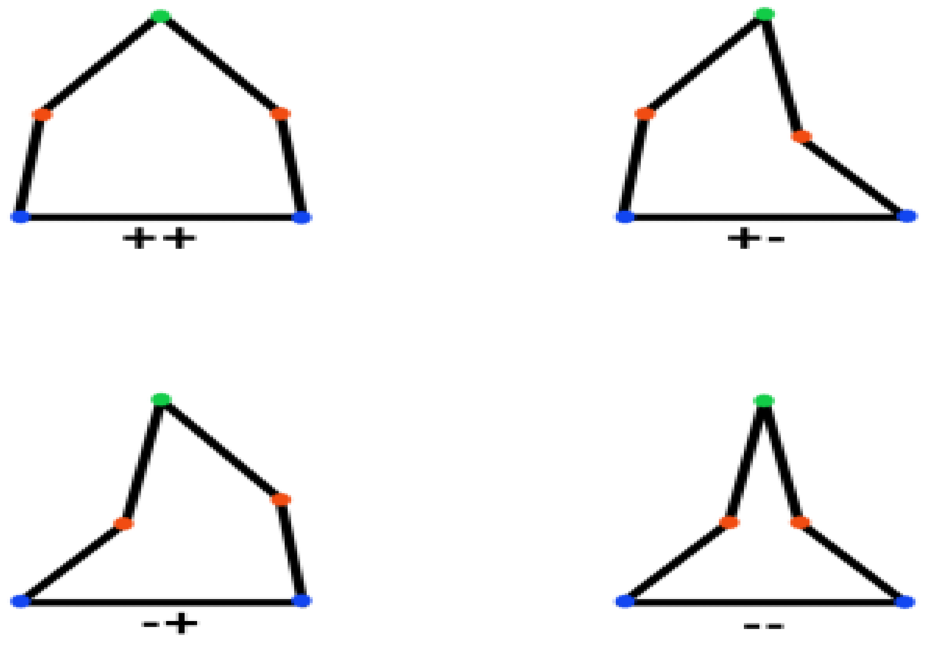
For the parallelogram linkage of Figure 2.7(a), $N = 5$, $J = 6$, and $f_i = 1$ for each joint. From Gr¨ubler's formula, the number of degrees of freedom is $3(5 - 1 - 6) + 6 = 0$. A mechanism with zero degrees of freedom is by definition a rigid structure.

It is clear from examining the figure, though, that the mechanism can in fact move with one degree of freedom

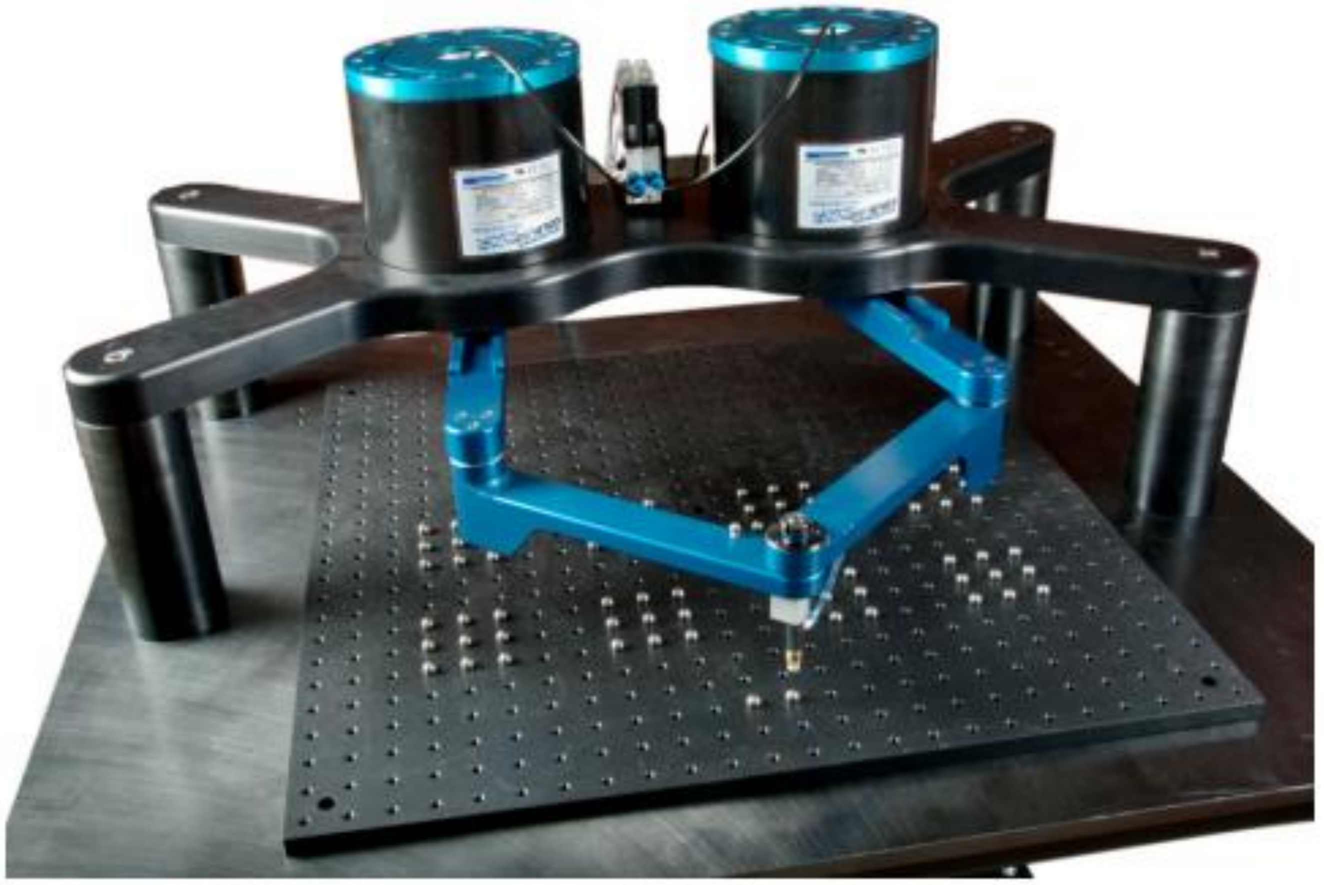


GRUBLER LAW FOR PLANAR ROBOTS

A similar situation arises for the two-dof planar five-bar linkage. If the two joints connected to ground are locked at some fixed angle, the five-bar linkage should then become a rigid structure.



The four Working Modes of 5BL.



DexTAR, a 5-bar linkage planar robot implementation[7]

GRUBLER LAW FOR SPATIAL ROBOTS

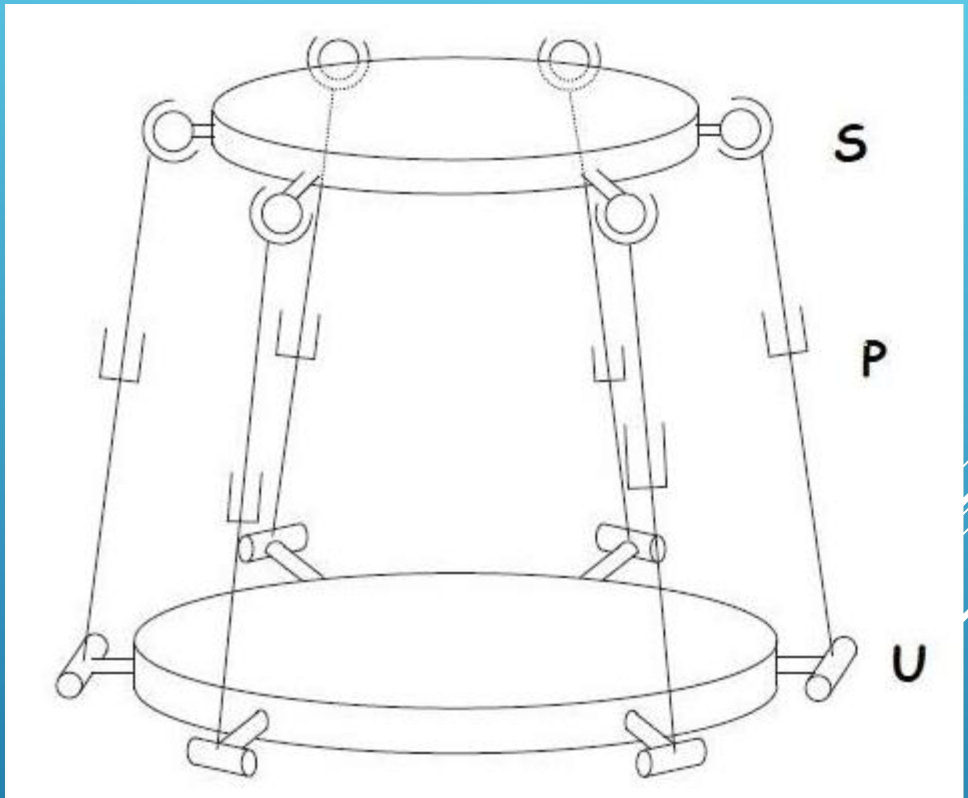
Proposition 2.2. *Consider a spatial robot consisting of* N *links (including ground),* J *joints (labelled from from 1 to* J*), and denote by* f_i *the degrees of freedom of joint* i*. The degrees of freedom (dof) of the robot can then be evaluated from the formula*

$$\begin{aligned} dof &= 6(N-1) - \sum_{i=1}^{J}(6-f_i) \\ &= 6(N-1-J) + \sum_{i=1}^{J} f_i. \end{aligned} \tag{2.11}$$

GRUBLER LAW FOR SPATIAL ROBOTS

Stewart-Gough Platform

N= 14
J=18
DOF=6(14-18-1)+6*3+6*1+6*2
DOF=-30+18+6+12
DOF=-30+36
DOF=6



GRUBLER LAW FOR SPATIAL ROBOTS

3 UPU Platform

N= 8

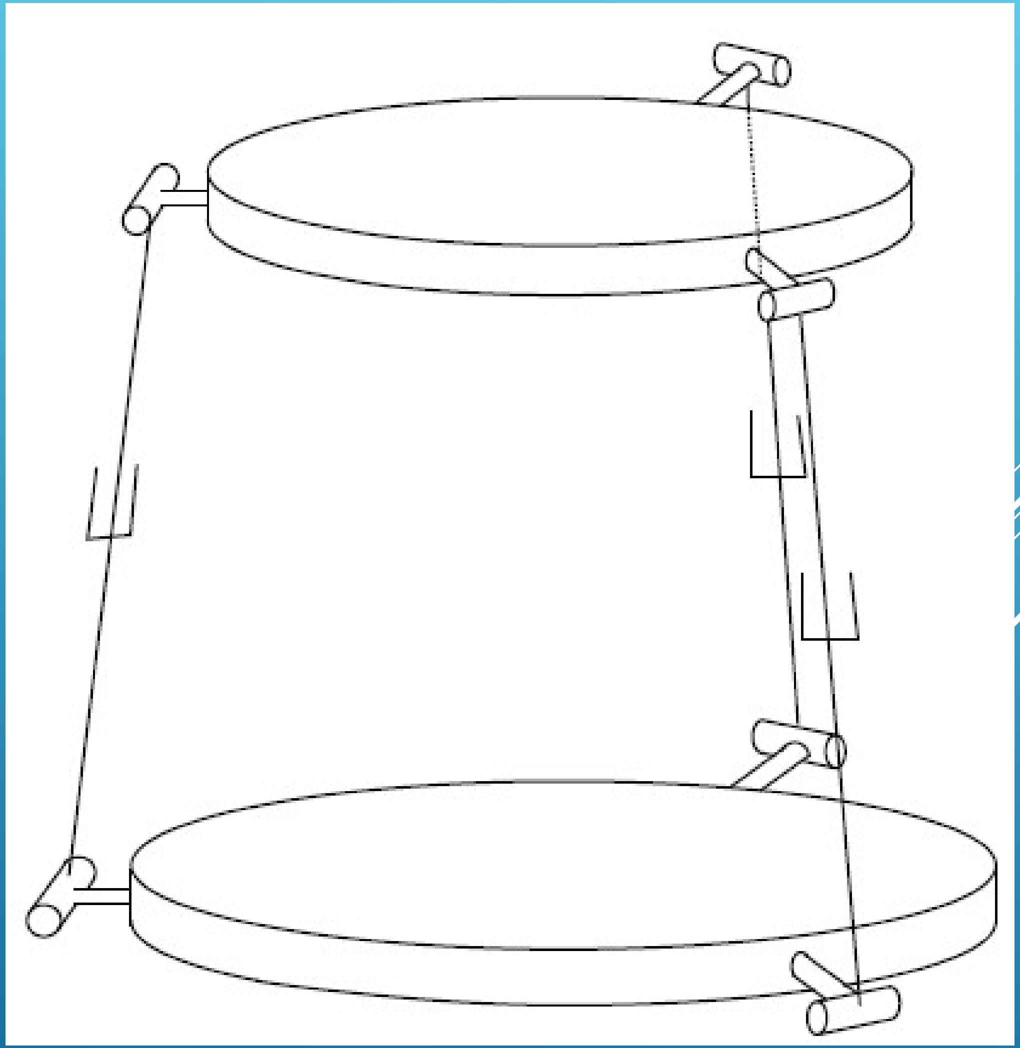
J=9

DOF=6(8-9-1)+3*2+3*1+3*2

DOF=-12+6+3+6

DOF=-12+15

DOF=3



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