

## (Biomedical Optics)

### 1- Plane-wave expansions

Plane-wave expansions are a mathematical and physical technique used to represent electromagnetic waves including light, as a superposition of plane waves in free space or homogeneous materials. This technique is based on the assumption that an electromagnetic wave can be represented as a sum of fundamental surface waves with a specific frequency, which propagate in all directions in free space or in the homogeneous material.

- a)** Plane-wave expansions are a powerful mathematical technique used to solve and analyze Maxwell's equations, and provide a complete description of electromagnetic waves in space (The propagation of electromagnetic waves can be described by Maxwell's equations).

$$1) \nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} , ( \mathbf{D} = \epsilon_0 \mathbf{E} ) , ( \nabla \cdot \mathbf{D} = \rho )$$

$$2) \nabla \cdot \mathbf{B} = 0$$

$$3) \nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}$$

$$4) ( \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} )$$

$$(\mathbf{B} = \mu_0 \mathbf{H})$$

$$\nabla \times \mathbf{B} = \mu_0 ( \mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} )$$

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Where:

**B, E, D** and **H** are the field vectors.

**J** is the current density.

**t** is the time.

**$\rho$**  is the charge density.

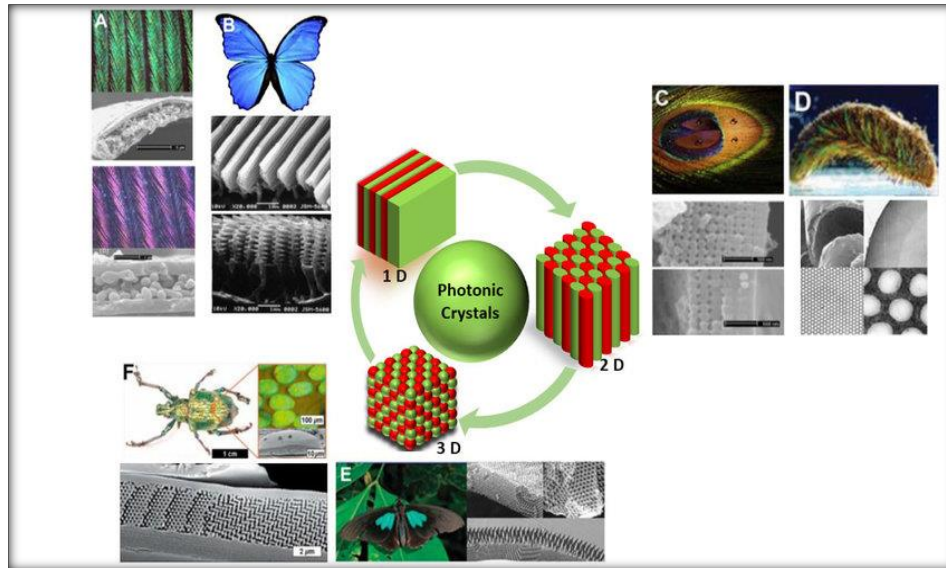
The constant is  $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2$  called the permittivity of free space.

The constant is  $\mu_0 = 4\pi \times 10^{-7} \text{ N} / \text{A}^2$  called the permeability of free space.

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**b)** Plane-wave expansions method is reliable method, which it is used to analyze the behavior of light within the periodic structure. Where, **Photonic crystals (PC)** are materials with a periodic structure that affects the motion of photons (light). The periodicity of the crystal structure leads to the formation of a photonic band gap a range of frequencies for which certain wavelengths of light are forbidden. Photonic crystals can control and manipulate the flow of light, and their properties have applications in various optical technologies. PC structures are common in nature, such as

in opals and butterfly wings, presenting as periodic nanostructures and bright structural colors.



Photonic crystals have been broadly used in many applications, including sensing, bio-detection, displays, optical devices, superconductor and solar cells.

## 2- Diffraction phenomenon

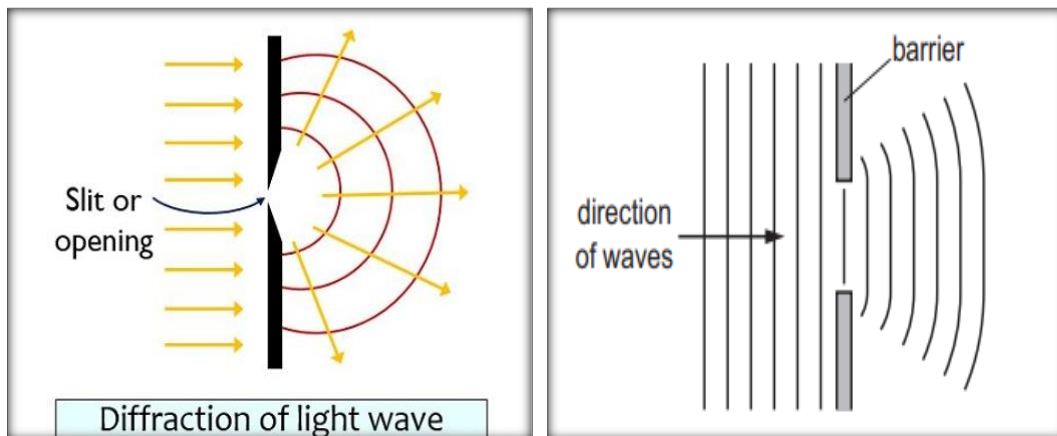
Diffraction in light refers to the bending and spreading of light waves as they pass through an opening or around a barrier. When light waves meet a small opening or a sharp edge, they tend to spread out in all directions, creating a pattern of interference and diffraction.

This pattern can be observed as bright and dark fringes on a screen placed behind the opening or obstacle. Diffraction of light is an important phenomenon that helps explain a wide range of optical phenomena,

including the formation of rainbows, the diffraction grating, and the interference pattern of double-slit experiments. It also plays a crucial role in the design and operation of many optical devices, such as telescopes, cameras, and microscopes.

**The amount of diffraction that occurs depends on:**

1. The size of the opening or obstacle.
2. The wavelength of the light.
3. The distance between the screen and the opening or obstacle.



### 3- The Rayleigh Criterion

The Rayleigh Criterion is a principle in optics that defines the minimum distance between two objects or points in an optical system to be able to distinguish them as separate. It sets a limit on the ability of optical systems, such as microscopes or telescopes, to resolve fine details. The criterion takes into account the wavelength of light and the size of the optical system's aperture, providing a guideline for designing systems with better resolution.

In simple terms, two points are considered just resolvable when the central bright spot of one point synchronizes with the first dark ring of the diffraction pattern of the other point.

The Rayleigh criterion for the minimum resolvable angle is:

where:

$$\theta = 1.22 \lambda/D$$

- $\theta$  is the angular resolution, or the smallest angle that can be resolved between two points in an image.
- $\lambda$  is the wavelength of light.
- $D$  is the diameter of the lens or aperture through which the light passes.

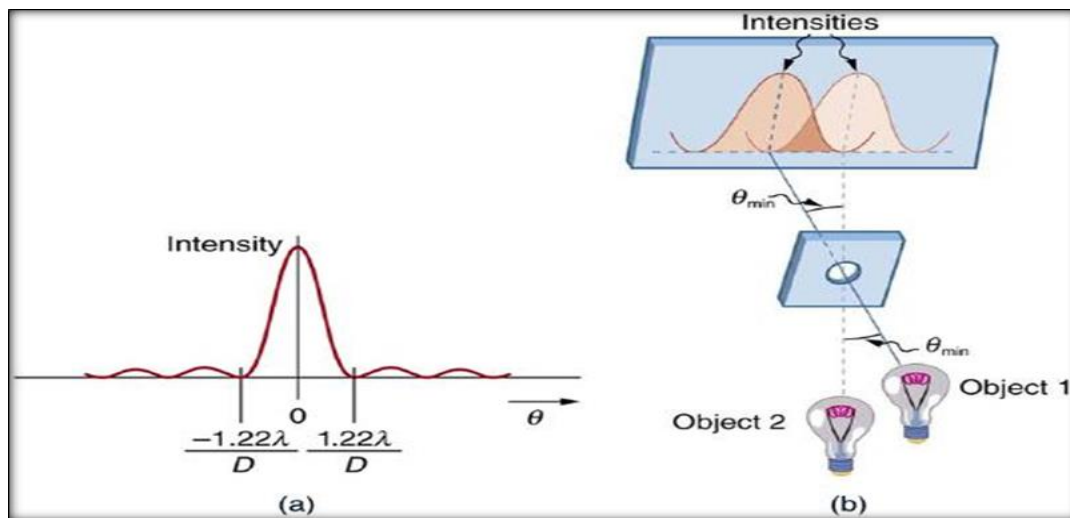


Figure: (a) Graph of intensity of the diffraction pattern for a circular aperture.

Note that, similar to a single slit, **the central maximum is wider and brighter than those to the sides.** (b) Two point objects produce overlapping diffraction patterns. Shown here is the Rayleigh criterion for being just resolvable. **The central maximum of one pattern lies on the first minimum of the other.**

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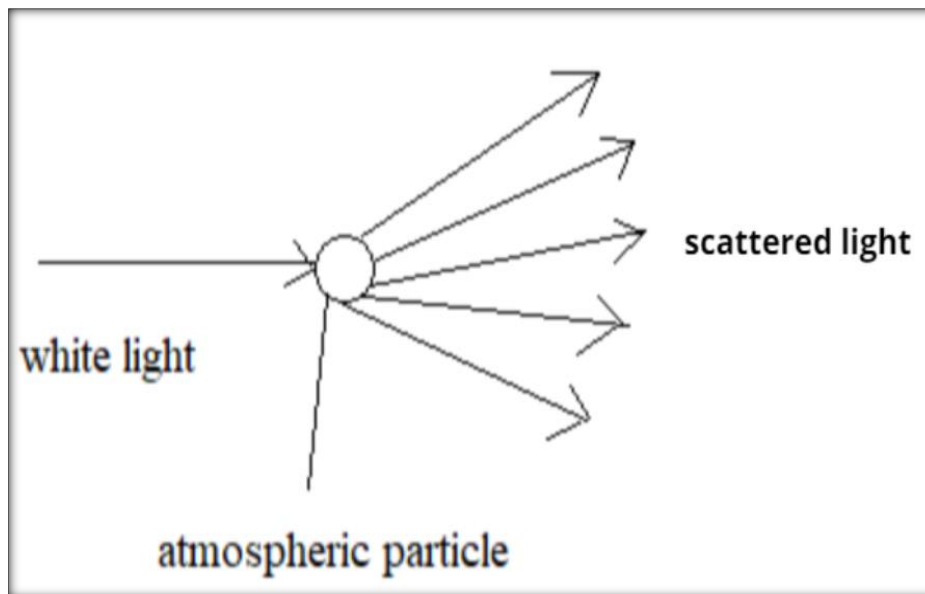
## 4-Scattering phenomenon

Scattering is a phenomenon that occurs when light interacts with matter and is redirected in different directions. When light passes through a medium, it interacts with the particles in that medium, such as molecules, atoms, or even small particles like dust or water droplets. These interactions cause the light to change direction and spread out, which is known as **scattering**.

1. **Rayleigh Scattering:** This type of scattering occurs when the size of the scattering particles is much smaller than the wavelength of the incident radiation. Rayleigh scattering is responsible for the blue color of the sky, as shorter wavelengths (blue light) are scattered more than longer wavelengths (red light) by the gases and small particles in the Earth's atmosphere.
2. **Mie Scattering:** occurs when the size of the scattering particles is similar to the wavelength of the incident radiation. It is prevalent in situations involving larger particles, such as water droplets in clouds, which can scatter light in various directions.
3. **Raman Scattering:** is a phenomenon where the incident light interacts with molecular vibrations in a material, leading to a shift in the frequency of the scattered light. This type of scattering provides information about the molecular composition of the material.
4. **Compton Scattering:** Involves the scattering of photons by electrons. It occurs when X-rays or gamma rays interact with electrons, leading

to a change in the direction and energy of the scattered radiation. Compton scattering is important in medical imaging and the study of atomic particles.

5. **Elastic and Inelastic Scattering:** Elastic scattering involves a change in direction without a change in energy, while inelastic scattering involves a change in both direction and energy of the scattered particles.
6. **Acoustic Scattering:** In acoustics, scattering occurs when sound waves encounter obstacles or irregularities in a medium. This phenomenon is relevant in underwater acoustics, sonar applications, and the study of material properties using ultrasound.



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Scattering phenomena have widespread applications, including remote sensing, atmospheric science, medical imaging, material characterization, and communication technologies.

## 5- The optical theorem

Is a fundamental principle in wave scattering theory that establishes a connection between the total scattering cross-section of a scatterer and the forward scattering amplitude. This theorem is widely applicable across various branches of physics, including optics, acoustics, and quantum mechanics. It is usually written in the form:

$$\sigma_{\text{tot}} = \frac{4\pi}{k} \text{Im } f(0),$$

Where:

$\sigma_{\text{tot}}$  is the total scattering cross-section.

$k$  is the wave number of the incident wave.

$\text{Im } f(0)$  is the scattering amplitude with an angle of zero, that is the amplitude of the wave scattered to the center of a distant screen.