INTRODUCTION TO CONTROL SYSTEMS

Dr.Alaa M. Al-Kaysi

LECTURE 1

4TH STAGE - 1ST SEMESTER - BIOMEDICAL INSTRUMENTATION AND BIOMECHANIC BRANCHES

Introduction to Control Systems

1. ***Controlled Variable and Control Signal or Manipulated Variable***

The *controlled* variable is the quantity or condition that is measured and controlled. The *control signal* or *manipulated* variable is the quantity or condition that is varied by the controller so as to affect the value of the controlled variable. Normally, the controlled variable is the output of the system. *Control* means measuring the value of the controlled variable of the system and applying the control signal to the system to correct or limit deviation of the measured value from a desired value. In studying control engineering, we need to define additional terms that are necessary to describe control systems.

2. ***Plants***

A plant may be a piece of equipment, perhaps just a set of machine parts functioning together, the purpose of which is to perform a particular operation, any physical object to be controlled (such as a mechanical device, a heating furnace, a chemical reactor, or a spacecraft) can be called a plant.

3. ***Processes***

The *Merriam–Webster Dictionary* defines a process to be a natural, progressively continuing operation or development marked by a series of gradual changes that succeed one another in a relatively fixed way and lead toward a particular result or end; or an artificial or voluntary, progressively continuing operation that consists of a series of controlled actions or movements systematically directed toward a particular result or end. any operation to be controlled can be called a *process*. Examples are chemical, economic, and biological processes.

4. ***Systems***

A system is a combination of components that act together and perform a certain objective. A system need not be physical. The concept of the system can be applied to abstract, dynamic phenomena such as those encountered in economics. The word system should, therefore, be interpreted to imply physical, biological, economic, and the like, systems.

5. ***Disturbances***

A disturbance is a signal that tends to adversely affect the value of the output of a system. If a disturbance is generated within the system, it is called *internal,* while an *external* disturbance is generated outside the system and is an input.

6. ***Feedback Control***

Feedback control refers to an operation that, in the presence of disturbances, tends to reduce the difference between the output of a system and some reference input and does so on the basis of this difference. Here only unpredictable disturbances are so specified, since predictable or known disturbances can always be compensated for within the system.

7. ***Robust Control System***

The first step in the design of a control system is to obtain a mathematical model of the plant or control object. In reality, any model of a plant we want to control will include an error in the modeling process. That is, the actual plant differs from the model to be used in the design of the control system.

8. ***Feedback Control Systems***

A system that maintains a prescribed relationship between the output and the reference input by comparing them and using the difference as a means of control is called a *feedback control system.* An example would be a room temperature control system. By measuring the actual room temperature and comparing it with the reference temperature (desired temperature), the thermostat turns the heating or cooling equipment on or off in such a way as to ensure that the room temperature remains at a comfortable level regardless of outside conditions.

Feedback control systems are not limited to engineering but can be found in various non-engineering fields as well. The human body, for instance, is a highly advanced feedback control system. Both body temperature and blood pressure are kept constant by means of physiological feedback. In fact, feedback performs a vital function: It makes the human body relatively insensitive to external disturbances, thus enabling it to function properly in a changing environment.

9. ***Closed-Loop Control Systems***

Feedback control systems are often referred to as *closed-loop control* systems. In practice, the terms feedback control and closed-loop control are used interchangeably. In a closed-loop control system the actuating error signal, which is the difference between the input signal and the feedback signal (which may be the output signal itself or a function of the output signal and its derivatives and/or integrals), is fed to the controller so as to reduce the error and bring the output of the system to a desired value. The term closed-loop control always implies the use of feedback control action in order to reduce system error.

10. ***Open-Loop Control Systems***

Those systems in which the output has no effect on the control action are called *open-loop control systems*. In other words, in an open loop control system the output is neither measured nor fed back for comparison with the input. One practical example is a washing machine. Soaking, washing, and rinsing in the washer operate on a time basis. The machine does not measure the output signal, that is, the cleanliness of the clothes. In any open-loop control system the output is not compared with the reference input. Thus, to each reference input there corresponds a fixed operating condition; as a result, the accuracy of the system depends on calibration. In the presence of disturbances, an open-loop control system will not perform the desired task. Open-loop control can be used, in practice, only if the relationship between the input and output is known and if there are neither internal nor external disturbances

The first step in control engineering is to understand the system that we want to control.

Then, we design a secondary system that controls the behavior of the first system

This is called ***open loop control***.

Examples of control systems:

1. Speed Control System

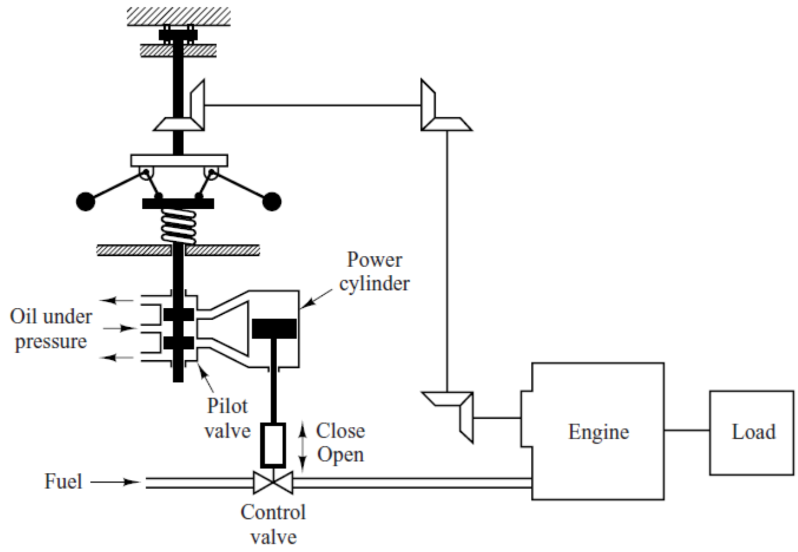

Figure (1): Speed control system

2. Temperature Control System

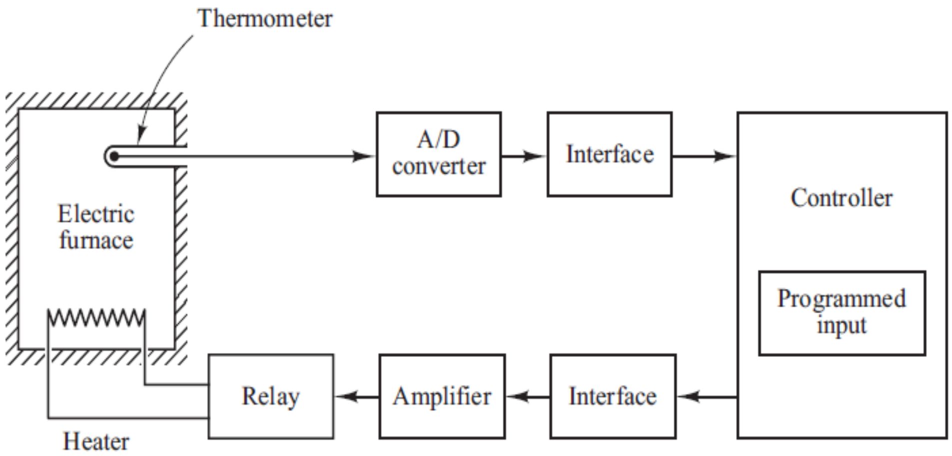

Figure (2): Temperature control system

Mathematical Modeling of Control Systems

Mathematical Models

Mathematical models may assume many different forms. Depending on the particular system and the particular circumstances, one mathematical model may be better suited than other models.

Linear Systems

A system is called linear if the principle of superposition applies. The principle of superposition states that the response produced by the simultaneous application of two different forcing functions is the sum of the two individual responses. Hence, for the linear system, the response to several inputs can be calculated by treating one input at a time and adding the results.

Linear Time-Invariant Systems and Linear Time-Varying Systems

A differential equation is linear if the coefficients are constants or functions only of the independent variable. Dynamic systems that are composed of linear time-invariant lumped-parameter components may be described by linear time-invariant differential equations—that is, constant-coefficient differential equations. Such systems are called *linear time-invariant* (or *linear constant-coefficient*) systems. Systems that are represented by differential equations whose coefficients are functions of time are called *linear time-varying* systems. An example of a time-varying control system is a spacecraft control system.

- **Linear time-invariant** (LTI) systems are composed of two types of systems, *linear* and *time-invariant*.
- A **time-invariant** (TI) system has the property that delaying the input by any constant D delays the output by the same amount:

Block Diagrams

A *block diagram* of a system is a pictorial representation of the functions performed by each component and of the flow of signals. Such a diagram depicts the inter-relationships that exist among the various components. Differing from a purely abstract mathematical representation, a block diagram has the advantage of indicating more realistically the signal flows of the actual system.

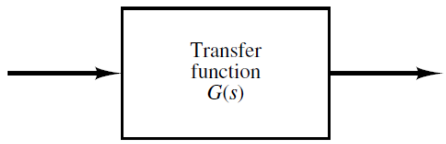

Figure (3): Element of a block diagram

Note that the dimension of the output signal from the block is the dimension of the input signal multiplied by the dimension of the transfer function in the block

Summing Point. Referring to Fig. (4), a circle with a cross is the symbol that indicates a summing operation. The plus or minus sign at each arrowhead indicates whether that signal is to be added or subtracted. It is important that the quantities being added or subtracted have the same dimensions and the same units.

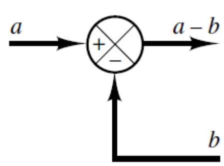

Figure (4): Summing point

Branch Point. A *branch point* is a point from which the signal from a block goes concurrently to other blocks or summing points.

Block Diagram of a Closed-Loop System

Figure 5 shows an example of a block diagram of a closed-loop system. The output $C(s)$ is fed back to the summing point, where it is compared with the reference input $R(s)$. The closed-loop nature of the system is clearly indicated by the figure. The output of the block, $C(s)$ in this case, is obtained by multiplying the transfer function $G(s)$ by the input to the block, $E(s)$. Any linear control system may be represented by a block diagram consisting of blocks, summing points, and branch points.

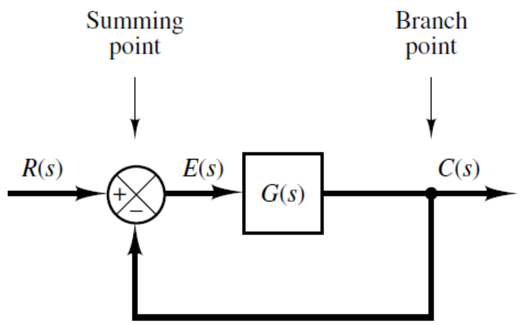

Figure (5): Block diagram of a closed-loop system

Transfer function and Impulse response function:

In control theory, functions called transfer functions are commonly used to characterize the input-output relationships of components or systems that can be described by linear, time-invariant, differential equations.

Transfer Function

The *transfer function* of a linear, time-invariant, differential equation system is defined as the *ratio of the Laplace transform of the output* (response function) *to the Laplace transform of the input* (driving function) *under the assumption that all initial conditions are zero*.

Consider the linear time-invariant system defined by the following differential equation:

$$a_0 \overset{(n)}{y} + a_1 \overset{(n-1)}{y} + \cdots + a_{n-1}\dot{y} + a_n y = b_0 \overset{(m)}{x} + b_1 \overset{(m-1)}{x} + \cdots + b_{m-1}\dot{x} + b_m x \qquad (n \geq m)$$

where y is the output of the system and x is the input. The transfer function of this system is the *ratio of the Laplace transformed output to the Laplace transformed input* when all initial conditions are **zero**, or

$$\text{Transfer function} = G(s) = \frac{\mathscr{L}[\text{output}]}{\mathscr{L}[\text{input}]}\bigg|_{\text{zero initial conditions}}$$

$$= \frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1}s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n}$$

By using the concept of transfer function, it is possible to represent system dynamics by algebraic equations in *s*. If the highest power of *s* in the denominator of the transfer function is equal to *n*, the system is called a *nth-order system*.

The applicability of the concept of the transfer function is limited to linear, time-invariant, differential equation systems. The transfer function approach, however, is extensively used in the analysis and design of such systems.

- The transfer function of a system is a mathematical model in that it is an operational method of expressing the differential equation that relates the output variable to the input variable.
- The transfer function is a property of a system itself, independent of the magnitude and nature of the input or driving function.
- The transfer function includes the units necessary to relate the input to the output; however, it does not provide any information concerning the physical structure of the system. (The transfer functions of many physically different systems can be identical.)
- If the transfer function of a system is known, the output or response can be studied for various forms of inputs with a view toward understanding the nature of the system.
- If the transfer function of a system is unknown, it may be established experimentally by introducing known inputs and studying the output of the system. Once established, a transfer function gives a full description of the dynamic characteristics of the system, as distinct from its physical description.

Convolution Integral. For a linear, time-invariant system the transfer $function\ G(s)$ is

$$G(s) = \frac{Y(s)}{X(s)}$$

where $X(s)$ is the Laplace transform of the input to the system and $Y(s)$ is the Laplace transform of the output of the system, where we assume that all initial conditions involved are zero. It follows that the output $Y(s)$ can be written as the product of $G(s)$ and $X(s)$, or

$$Y(s) = G(s)X(s) \tag{1}$$

Note: the multiplication in the complex domain is equivalent to convolution in the time domain, so the inverse Laplace transform of Equation (1) is given by the following *convolution integral*:

$$y(t) = \int_0^t x(\tau)g(t-\tau)d\tau$$
$$= \int_0^t g(\tau)x(t-\tau)d\tau$$

where both $g(t)$ and $x(t)$ are 0 for $t < 0$.

Impulse-Response Function

Consider the output (response) of a linear time invariant system to a unit-impulse input when the initial conditions are zero. Since the Laplace transform of the unit-impulse function is unity, the Laplace transform of the output of the system is

$$Y(s) = G(s) \tag{2}$$

The unit impulse signal, $\delta(t)$, however is more difficult to define than the unit impulse sequence, $\delta[n]$

Recall that

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

The unit impulse signal is defined as

$$\delta(t) = 0,\ t \neq 0 \quad \text{and} \quad \int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

It would seem that $\delta(t)$ must have zero width, yet have area of unity