# BLOCK DIAGRAM

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**LECTURE 2**

4TH STAGE - 1ST SEMESTER - BIOMEDICAL INSTRUMENTATION AND BIOMECHANIC BRANCHES

# Lecture 2: Block Diagram

Block diagram is a shorthand, graphical representation of a physical system, illustrating the functional relationships among its components.

The simplest form of the block diagram is the single block, with one input and one output.


$$C(s) = G(s)R(s)$$

Figure (6): Single block diagram representation [Control and Instrumentation-Ref]

In particular, a block diagram of a system is a pictorial representation of the functions performed by each component of the system and shows the flow of signals. In block diagram, the system consists of so many components. These components are linked together to perform a particular function. Each component can be represented with the help of individual block.

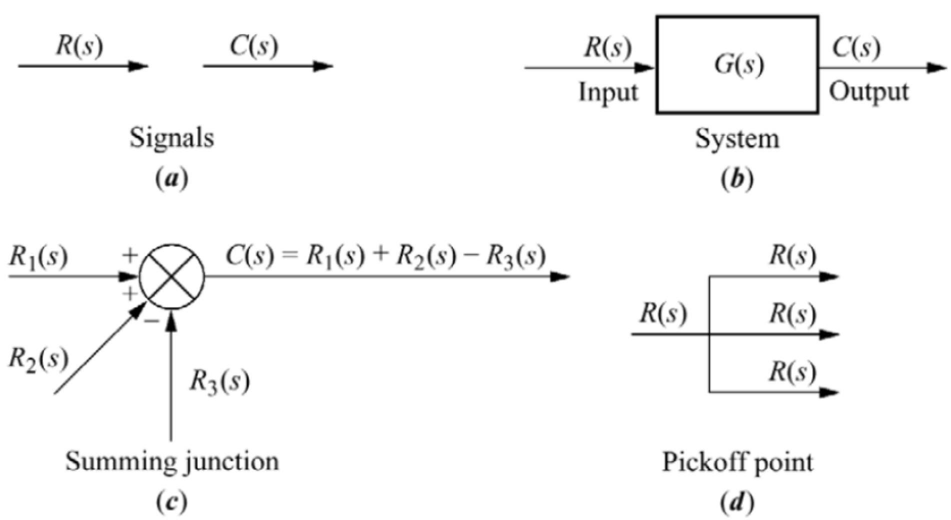



Figure (7): Components of Linear Time Invariant Systems (LTIS)

**Open-Loop Transfer Function and Feedforward Transfer Function.**

Referring to Figure 8, the ratio of the feedback signal B(s) to the actuating error signal E(s) is called the open-loop transfer function. That is,

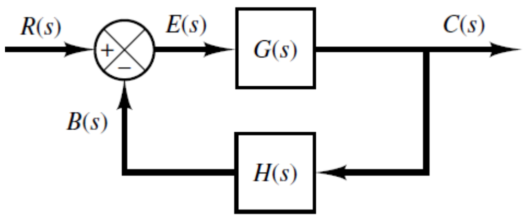



Figure (8)

$$\textit{Open-loop transfer function} = \frac{B(s)}{E(s)} = G(s)H(s)$$

The ratio of the output $C(s)$ to the actuating error signal $E(s)$ is called the feedforward transfer function, so that

$$\textit{Feedforward transfer function} = \frac{C(s)}{E(s)} = G(s)$$

If the feedback transfer function $H(s)$ is unity, then the open-loop transfer function and the feedforward transfer function is the same.

**Closed-Loop Transfer Function.**

For the system shown in Figure 8, the output $C(s)$ and input $R(s)$ are related as follows: since

$$\begin{aligned} C(s) &= G(s)E(s) \\ E(s) &= R(s) - B(s) \\ &= R(s) - H(s)C(s) \end{aligned}$$

eliminating E(s) from these equations gives

$$C(s) = G(s)\left[R(s) - H(s)C(s)\right]$$

or

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \tag{3}$$

The transfer function relating C(s) to R(s) is called the closed-loop transfer function. It relates the closed-loop system dynamics to the dynamics of the feedforward elements and feedback elements. From Equation (3), $C(s)$ is given by

$$C(s) = \frac{G(s)}{1 + G(s)H(s)} R(s)$$

Thus the output of the closed-loop system clearly depends on both the closed-loop transfer function and the nature of the input

**Obtaining Cascaded, Parallel, and Feedback (Closed-Loop) Transfer Functions**

In control-systems analysis, we frequently need to calculate the cascaded transfer functions, parallel-connected transfer functions, and feedback-connected (closed-loop) transfer functions. Suppose that there are two components G1(s) and G2(s) connected differently as shown in Figure 9: (a), (b), and (c).

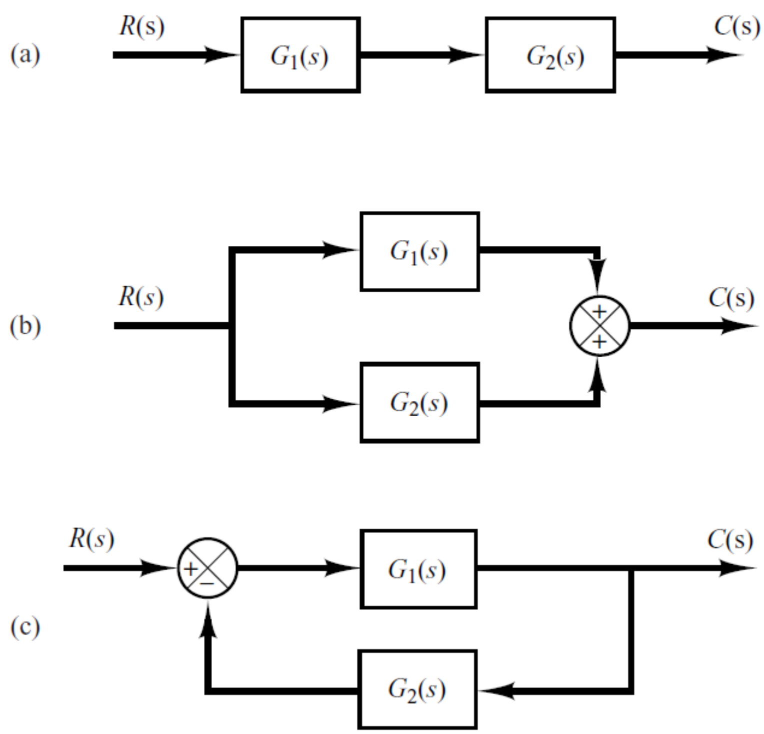



Figure 9: (a) Cascaded system; (b) parallel system; (c) feedback (closed-loop) system.

## Block Diagrams Reduction:

According to the control system shown in Fig. 9:

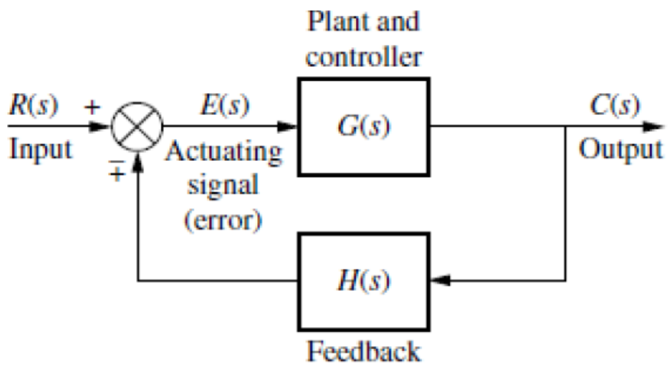



Figure 10: Block diagram of a closed-loop system with a feedback element

- $G(s) = Direct\ transfer\ function = forward\ transfer\ function$
- $H(s) = feedback\ transfer\ function$
- $G(s)H(s) = open - loop\ transfer\ function$
- $C(s)/R(s) = closed - loop\ transfer\ function$
- $C(s)/E(s) = feed - forward\ transfer\ function$

The steps to reduce the block diagram:

- ✓ Reduce the series blocks.
- ✓ Reduce the parallel blocks
- ✓ Reduce minor feedback loops.
- ✓ As for as possible shift summing point to the left and take-off point to the right.
- ✓ Repeat the above steps till canonical form is obtained.

**1.** Cascade (Series) Connection

Figure 11-(a) shows an example of cascaded subsystems. Intermediate signal values are shown at the output of each subsystem. Each signal is derived from the product of the input times the transfer function. The equivalent transfer function shown in Fig. 11-(b), is the output Laplace

transform divided by the input Laplace which is the product of the subsystems' transfer functions

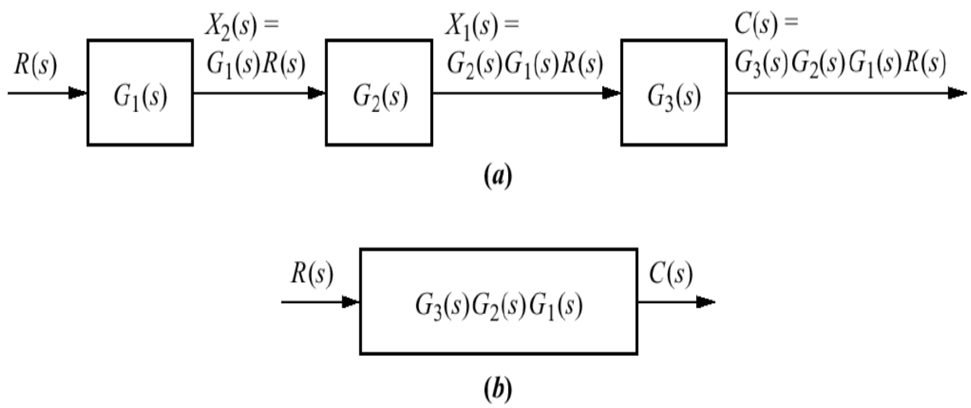



Figure 12: Cascaded system (a) Original Block Diagram (b) Equivalent Block Diagram

**2.** Parallel Connection

Figure 12- (a) shows an example of parallel subsystems. Again, by writing the output of each subsystem, we can find the equivalent transfer function. Parallel subsystems have a common input and an output formed by the algebraic sum of the outputs from all of the subsystems. The equivalent transfer function is given in Fig. 12-(b):

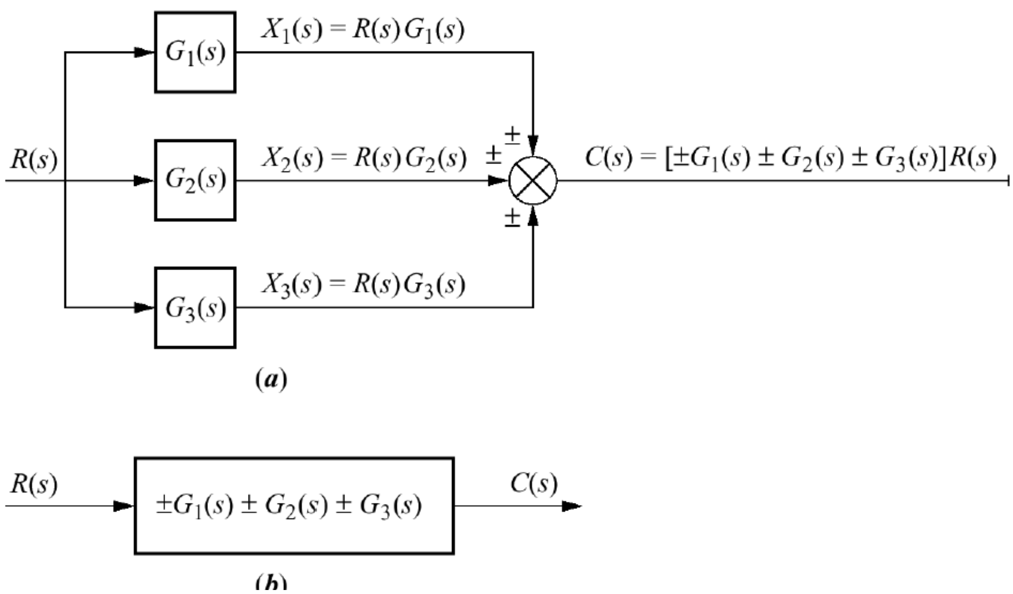



Figure 13: Parallel system (a) Original Block Diagram (b) Equivalent Block Diagram

**3.** Feedback Connections

The third connection is the feedback form as shown in Fig. 14 (a). The feedback forms the basis for our study of control systems engineering.

*We know that* $C(s) = G(s)\,E(s)$ & $B(s) = H(s)C(s)$

Where $E(s) = R(s) \mp B(s) = R(s) \mp H(s)C(s)$

Eliminating $E(s)$ from these equations gives

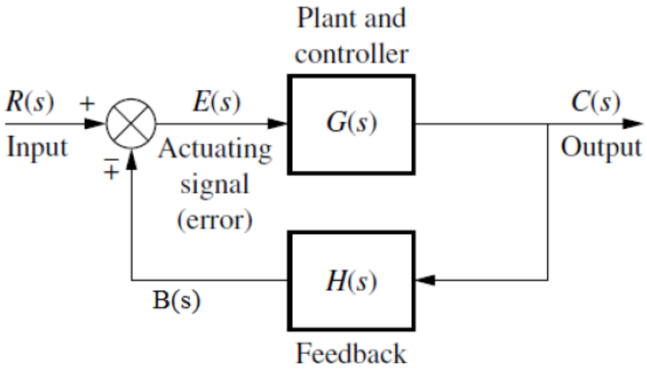
$C(s) = G(s)E(s) \Rightarrow C(s) = G(s)[R(s) \mp H(S)C(S)]$ *This can be written in the form*

$C(s) \pm G(s)H(s)C(s) = G(s)R(s)$

$[1 \pm G(s)H(s)]C(s) = G(s)R(s)$

$$\therefore \frac{\boldsymbol{C(s)}}{\boldsymbol{R(s)}} = \frac{\boldsymbol{G(s)}}{\boldsymbol{1 \pm G(s)H(s)}}$$

The equivalent transfer function is given in Fig. 14(b):



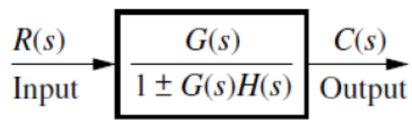



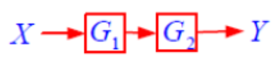
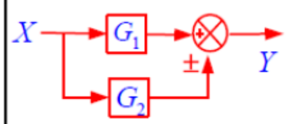
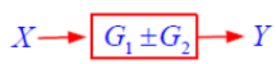
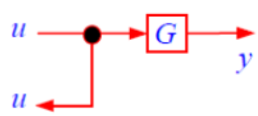
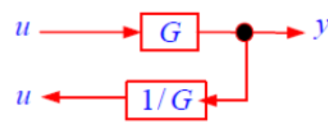
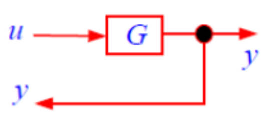
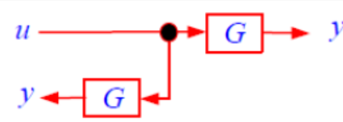
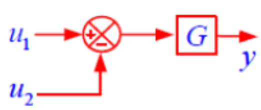
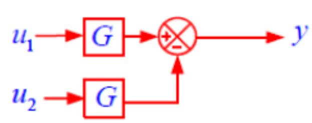
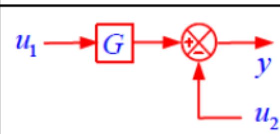
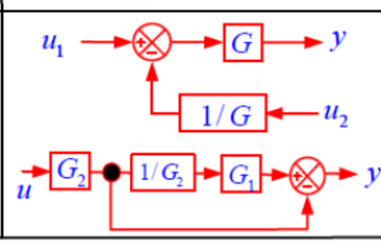

Figure 14: (a) Feedback connection (b) Equivalent block

The ***Characteristic equation*** of the system is defined as (*an equation obtained by setting the denominator polynomial of the transfer function to zero*).

The Characteristic equation for the above system is:

$$\boldsymbol{1 + G(s)H(s) = 0}$$

Table 1: Basic rules with block diagram transformation [Control and Instrumentation-Ref]

| | Manipulation | Original Block Diagram | Equivalent Block Diagram | Equation |
|---|---|---|---|---|
| 1 | Combining Blocks in Cascade |  |  | $Y=(G_1G_2)X$ |
| 2 | Combining Blocks in Parallel; or Eliminating a Forward Loop |  |  | $Y=(G_1\pm G_2)X$ |
| 3 | Moving a pickoff point behind a block |  |  | $y = G\,u$<br>$u = \frac{1}{G}\,y$ |
| 4 | Moving a pickoff point ahead of a block |  |  | $y=Gu$ |
| 5 | Moving a summing point behind a block |  |  | $e_2=G(u_1-u_2)$ |
| 6 | Moving a summing point ahead of a block |  |  | $y=Gu_1-u_2$<br>$y=(G_1-G_2)u$ |