University of Technology

Biomedical Engineering Department

Electrical Circuits

First Sage

Lecture (2)

By

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Introduction to Electrical Circuits

1.2.3 Kirchhoff's law

Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

Mathematically, KCL implies that:

$$\sum_{n=1}^{N} i_n = 0$$

where N is the number of branches connected to the node and i_n is the nth current entering (or leaving) the node.

The sum of the currents entering a node is equal to the sum of the cur-rents leaving the node.

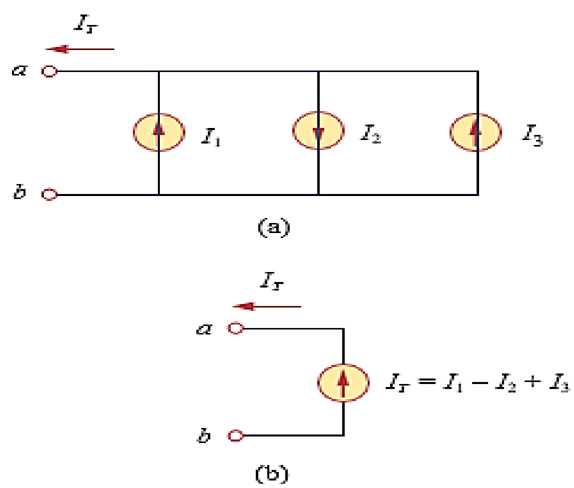


Figure (2.3): Current sources in parallel: (a) original circuit, (b) equivalent circuit.

Kirchhoff's voltage law (KVL) states that the algebraic sum of all volt-ages around a closed path (or loop) is zero.

Expressed mathematically, KVL states that

$$\sum_{m=1}^{M} v_m = 0$$

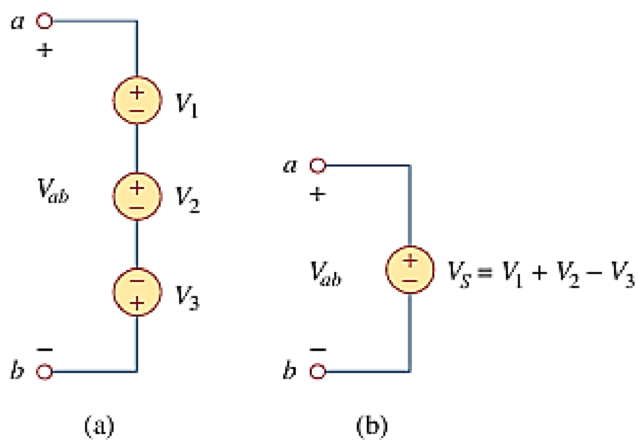


Figure (2.4): Voltage sources in series: (a) original circuit, (b) equivalent circuit.

Example (1) : For the circuit in Fig. 2.5 , find voltages v_1 and v_2.

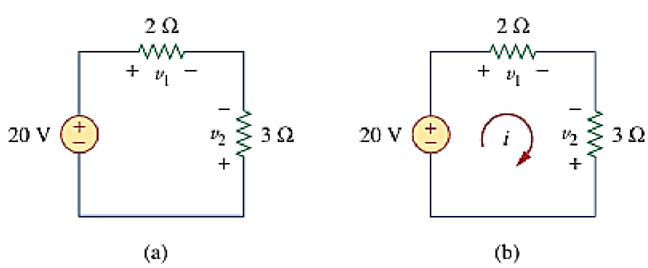


Figure: (2.5)

Solution :

To find v_1 and v_2, we apply Ohm's law and Kirchhoff's voltage

law. Assume that current i flows through the loop as shown in Fig. (2.5).
From Ohm's law.

v1 = 2I, v2 = -3I ………………………………………………….(1)

Applying KVL around the loop gives

-20 + v1 - v2 = 0 ………………………………………………….(2)

Substituting Eq. (1) into Eq. (2), we obtain

-20 + 2I + 3I = 0 or ► 5I= 20 I = 4 A

Substituting I in Eq. (1) finally gives

v1 = 8 V, v2 = -12 V

Example (2) :Find currents and voltages in the circuit shown in Fig. (2.6).

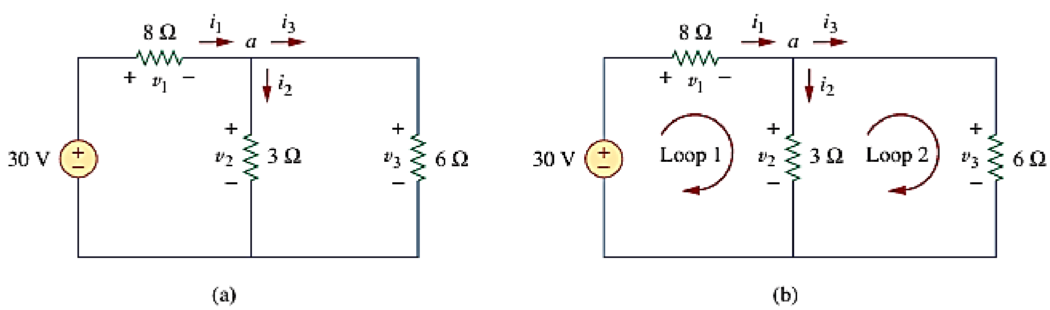


Figure : (2.6)

Solution:

We apply Ohm's law and Kirchhoff's laws. By Ohm's law, $v_1 = 8i_1, v_2 = 3i_2, v_3 = 6i_3$

Since the voltage and current of each resistor are related by Ohm's law as shown, we are really looking for three things: (v_1, v_2, v_3) or (i_1, i_2, i_3). At node a, KCL gives

$$i_1 - i_2 - i_3 = 0$$

Applying KVL to loop 1 as in Fig. 2.27(b),

$$-30 + v_1 + v_2 = 0$$

$$-30 + 8i_1 + 3i_2 = 0$$

or

$$i_1 = \frac{(30 - 3i_2)}{8}$$

Applying KVL to loop 2,

$$-v_2 + v_3 = 0 \quad \Rightarrow \quad v_3 = v_2$$

$$6i_3 = 3i_2 \quad \Rightarrow \quad i_3 = \frac{i_2}{2}$$

$$\frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0 \qquad\qquad i_2 = 2 \text{ A.}$$

$$i_1 = 3\text{ A}, \quad i_3 = 1\text{ A}, \quad v_1 = 24\text{ V}, \quad v_2 = 6\text{ V}, \quad v_3 = 6\text{ V}$$

Example (3): Determine V_o and I in the circuit shown in Fig. (7).

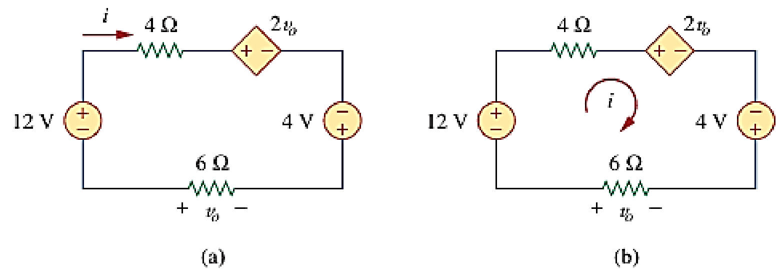


Figure (7)

Solution:

$$-12 + 4i + 2v_o - 4 + 6i = 0$$

Applying Ohm's law to the 6-Ω resistor gives

$$v_o = -6i$$

$$-16 + 10i - 12i = 0 \quad \Rightarrow \quad i = -8 \text{ A}$$

and $v_o = 48$ V.

Example (4): Find current I_o and voltage V_o in the circuit shown in Fig.(8).

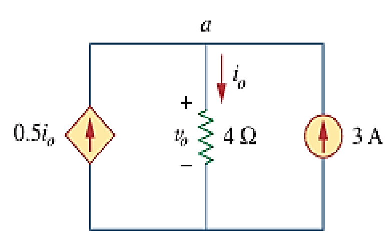


Figure (8)

Solution:

Applying KCL to node a, we obtain

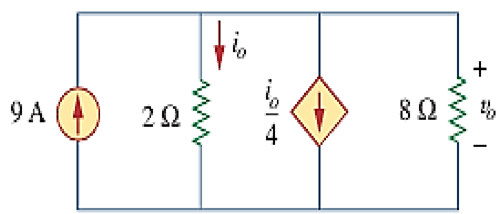
$$3 + 0.5i_o = i_o \quad \Rightarrow \quad i_o = 6 \text{ A}$$

For the 4-Ω resistor, Ohm's law gives

$$v_o = 4i_o = 24 \text{ V}$$

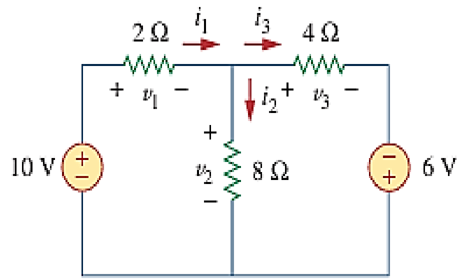
Problems

1. **Find the Io and Vo in the figure bellow:**



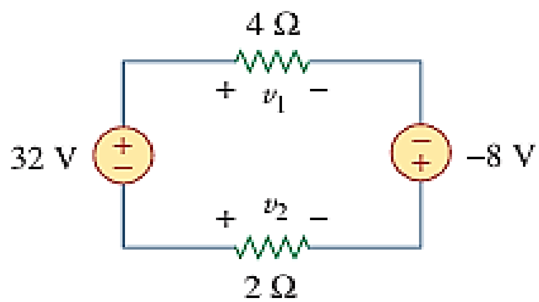
Answer: 12 V, 6 A.

2. **Find the voltages and currents in the figure bellow:**



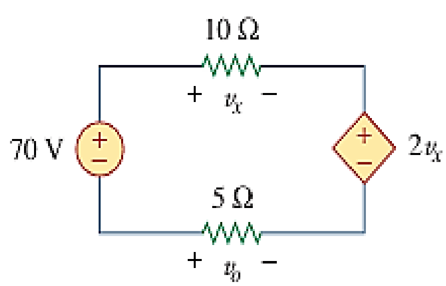
Answer: $v_1 = 6$ V, $v_2 = 4$ V, $v_3 = 10$ V, $i_1 = 3$ A, $i_2 = 500$ mA, $i_3 = 1.25$ A.

3. **Find V1 and V2 in the figure bellow:**



Answer: 16 V, −8 V.

4. **Find Vx and Vo in the figure bellow:**



Answer: 20 V, −10 V.

2. Circuits Transformations

2.1 Series Resistors and Voltage Division

The two resistors are in series, since the same current *I* flow in both. Applying Ohm's law to each of the resistors, we obtain:

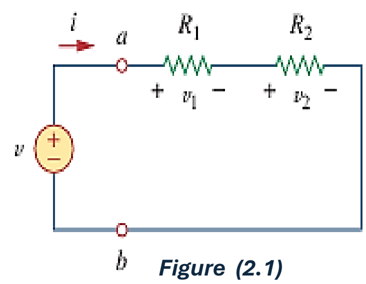


Figure (2.1)

$$v_1 = iR_1, \qquad v_2 = iR_2$$

If we apply KVL to the loop (moving in the clockwise direction), we have: $-V+V_1+V_2=0$

by combination the equations above, we get:

$$i = \frac{v}{R_1 + R_2}$$

$$v = iR_{eq}$$

$$R_{eq} = R_1 + R_2$$

The equivalent resistance of any number of resistors connected inseries is the sum of the individual resistances.

To determine the voltage across each resistor in Figure (2.1):

$$v_1 = \frac{R_1}{R_1 + R_2}v, \qquad v_2 = \frac{R_2}{R_1 + R_2}v$$

The equation states the ***voltage divider rule*** , which is means that:

The voltage across a resistor in a series circuit is equal to the value of that resistor times the total applied voltage across the series element divided by the total resistance of the series element .

2.2 Parallel Resistors and Current Division

In figure (2.2), we have two resistors in parallel and therefore have the same voltage across them. From ohm's law :

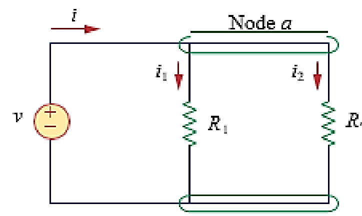


Figure (2.2)

$$v = i_1 R_1 = i_2 R_2$$

$$i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2}$$

Applying KCL at node a we get the total current I:

$$i = i_1 + i_2$$

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

or

$$\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$$

or

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.

The equivalent conductance of resistors connected in parallel is the sum of their individual conductance.

$$G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N$$

The equivalent conductance of resistors connected in series is obtained as :

$$\frac{1}{G_{eq}} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3} + \cdots + \frac{1}{G_N}$$

For the parallel circuits the current divider rule is:

$$i_1 = \frac{R_2 i}{R_1 + R_2}, \qquad i_2 = \frac{R_1 i}{R_1 + R_2}$$

Example 1 : For the circuit shown in figure (2.3) find the R_{eq}:

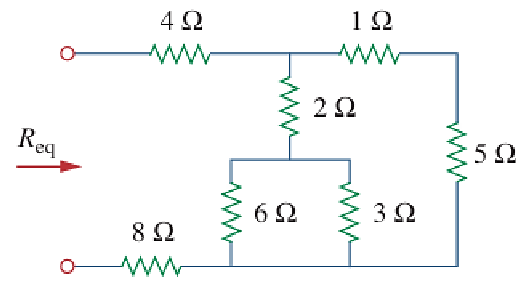


Figure: (2.3)

Solution:

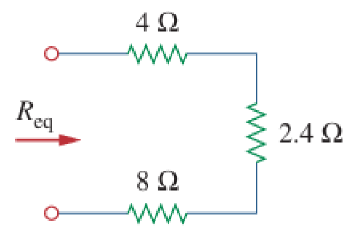
$$6\ \Omega \parallel 3\Omega = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$

$$1\ \Omega + 5\ \Omega = 6\ \Omega$$

$$2\ \Omega + 2\ \Omega = 4\ \Omega$$

$$4\ \Omega \parallel 6\ \Omega = \frac{4 \times 6}{4 + 6} = 2.4\ \Omega$$

$$R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega$$



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Example 2 : For the circuit shown in figure (2.4) find the R_{ab}

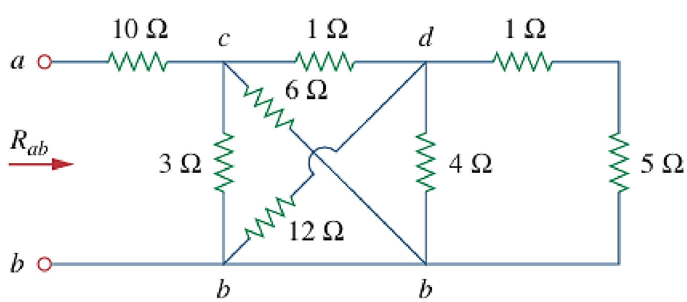

Figure:(2.4)

$$3\ \Omega \parallel 6\ \Omega = \frac{3 \times 6}{3 + 6} = 2\ \Omega$$

$$12\ \Omega \parallel 4\ \Omega = \frac{12 \times 4}{12 + 4} = 3\ \Omega$$

$$1\,\Omega + 5\,\Omega = 6\,\Omega$$

$$2\,\Omega \parallel 3\,\Omega = \frac{2 \times 3}{2 + 3} = 1.2\,\Omega$$

$$R_{ab} = 10 + 1.2 = 11.2\,\Omega$$

Example 3: find Vo and Io in figure (2.5)

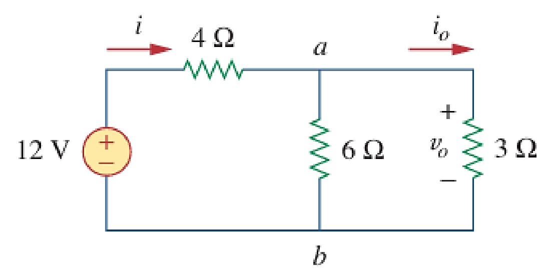

Figure:(2.5)

$$6\,\Omega \parallel 3\,\Omega = \frac{6 \times 3}{6 + 3} = 2\,\Omega$$

$$i = \frac{12}{4 + 2} = 2\text{ A}$$

$$v_o = \frac{2}{2 + 4}(12\text{ V}) = 4\text{ V}$$

$$v_o = 3i_o = 4 \quad \Rightarrow \quad i_o = \frac{4}{3}\text{ A} \qquad i_o = \frac{6}{6 + 3}i = \frac{2}{3}(2\text{ A}) = \frac{4}{3}\text{ A}$$

The power dissipated in the 3-Ω resistor is

$$p_o = v_o i_o = 4\left(\frac{4}{3}\right) = 5.333\text{ W}$$

Example 4 : For the circuit shown in Figure (2.6),(a), determine: (a) the voltage V_o, (b) the power supplied by the current source, (c) the power absorbed by each resistor.

(a)The 6-kΩ and 12-kΩ resistors are in series so that their combined value is 6 + 12 = 18 kΩ. Thus, the circuit in Fig.(2.)(a) reduces to that shown in Fig. (2.6)(b). We now apply the current division techniqueto find i_1 and i_2.

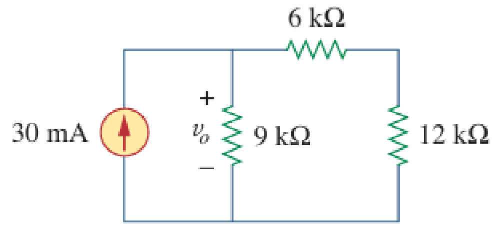

Figure:(2.6)

$$i_1 = \frac{18{,}000}{9{,}000 + 18{,}000}(30 \text{ mA}) = 20 \text{ mA}$$

$$i_2 = \frac{9{,}000}{9{,}000 + 18{,}000}(30 \text{ mA}) = 10 \text{ mA}$$

$$p_o = v_o i_o = 180(30) \text{ mW} = 5.4 \text{ W}$$

(b) Power supplied by the source is

$$p_o = v_o i_o = 180(30) \text{ mW} = 5.4 \text{ W}$$

(c) Power absorbed by the 12-kΩ resistor is

$$p = iv = i_2(i_2 R) = i_2^2 R = (10 \times 10^{-3})^2 (12{,}000) = 1.2 \text{ W}$$

Power absorbed by the 6-kΩ resistor is

$$p = i_2^2 R = (10 \times 10^{-3})^2 (6{,}000) = 0.6 \text{ W}$$

Power absorbed by the 9-kΩ resistor is

$$p = \frac{v_o^2}{R} = \frac{(180)^2}{9{,}000} = 3.6 \text{ W}$$

or

$$p = v_o i_1 = 180(20) \text{ mW} = 3.6 \text{ W}$$

Problems

Q1: For the circuit shown in figure (2.7) find the R_{ab}:

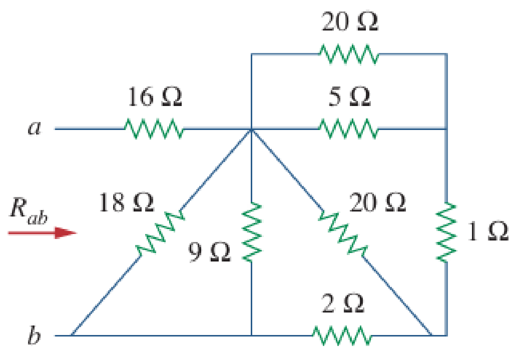

Figure:(2.7)

Ans: 19 ohms

Q2: For the circuit shown in figure (2.8) find the R_{ab}:

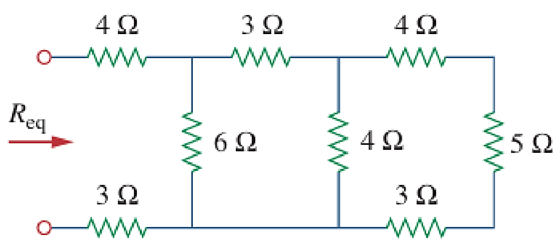

Figure: (2.8)

Ans :10 ohms

Q3: for the circuit shown in figure (2.9) find V1,V2.I1, I2 and the power dissipated in 12-ohm and 40-ohm resistors .

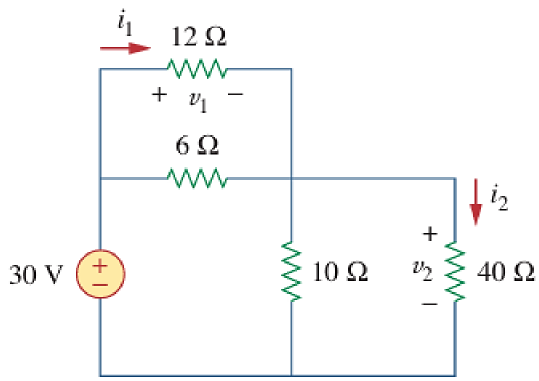

Figure 1:(2.9)

Answer: $v_1 = 10$ V, $i_1 = 833.3$ mA, $p_1 = 8.333$ W, $v_2 = 20$ V, $i_2 = 500$ mA, $p_2 = 10$ W.

Q4 : For the circuit shown in Fig. (2.10), find: (a) v_1 and v_2, (b) the power dissipated in the 3-kΩ and 20-kΩ resistors, and (c) the power supplied by the current source.

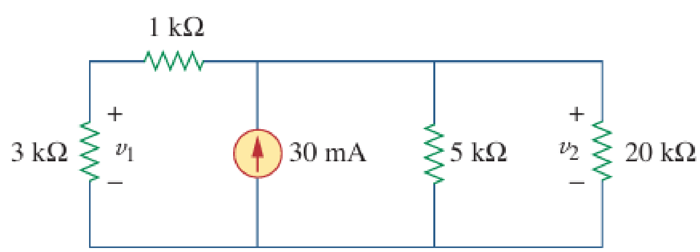

Figure:(2.10)

Answer: (a) 45 V, 60 V, (b) 675 mW, 180 mW, (c) 1.8 W.