University of Technology

Biomedical Engineering Department

Electrical Circuits

First Sage

Lecture (3)

By

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3.Star-Delta and Delta-Star Transformations

How do we combine resistors R_1 through R_6 when the resistors are neither in series nor in parallel? Many circuits of the type shown in Fig. 3.1 can be simplified by using three-terminal equivalent networks. These are the wye (Y), or tee (T) network shown in Fig.(3.2) and the delta (Δ), or pi (π) network shown in Fig. (3.3). These networks occur by themselves or as part of a larger network. They are used in three-phase networks, electrical filters, and matching networks. Our main interest here is in how to identify them when they occur as part of a network and how to apply wye-delta transformation in the analysis of that network.

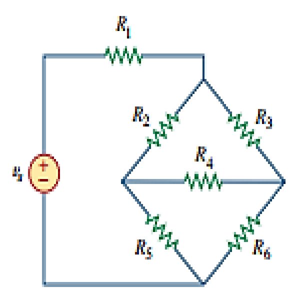


Figure : (3.1)

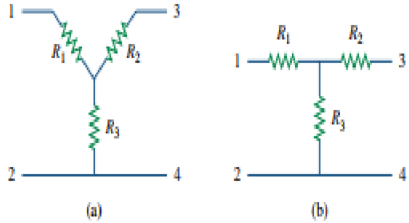


Figure (3.2):Two forms of the same network: (a) Y, (b) T.

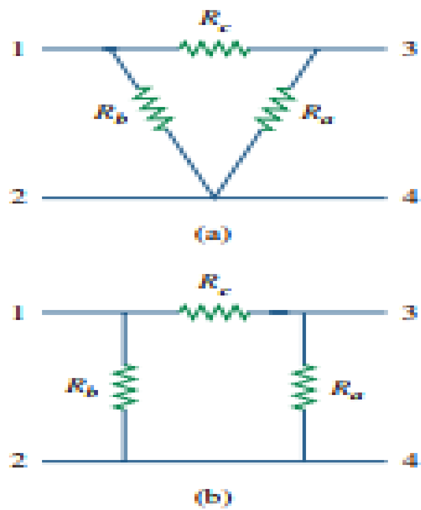


Figure (3.3): Two forms of the same network: (a) :Δ,(b):π.

3.1 Delta -Star transformation

Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

3.2 Star-Delta Transformations

Each resistor in the Δ network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

The Y and Δ networks are said to be *balanced* when

$$R_1 = R_2 = R_3 = R_Y, \qquad R_a = R_b = R_c = R_\Delta$$

Under these conditions, conversion formulas become

$$R_Y = \frac{R_\Delta}{3} \qquad \text{or} \qquad R_\Delta = 3R_Y$$

Example (1): convert the Δ in fig.(3.4) to an equivalent Y :

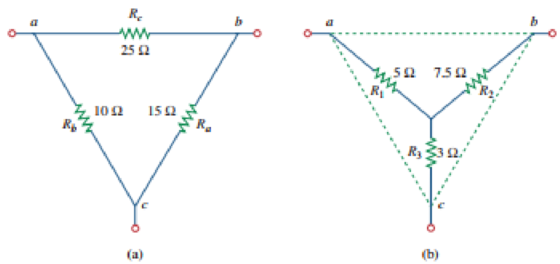


Figure (3.4): (a) original Δ network, (b) Y equivalent network.

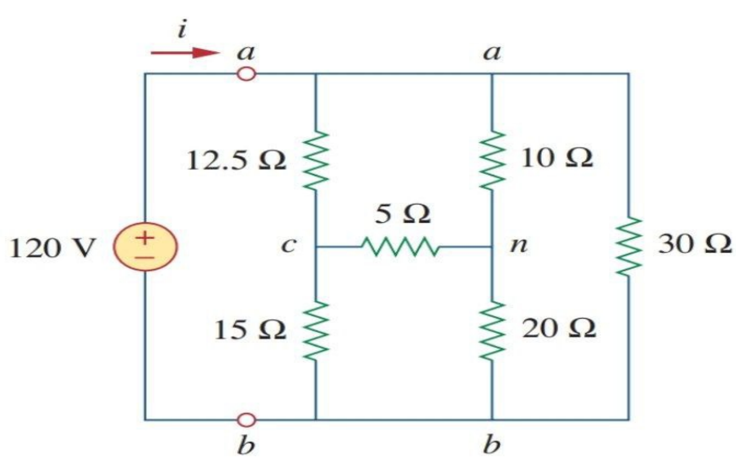
Solution:

$$R_1 = \frac{R_bR_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5\ \Omega$$

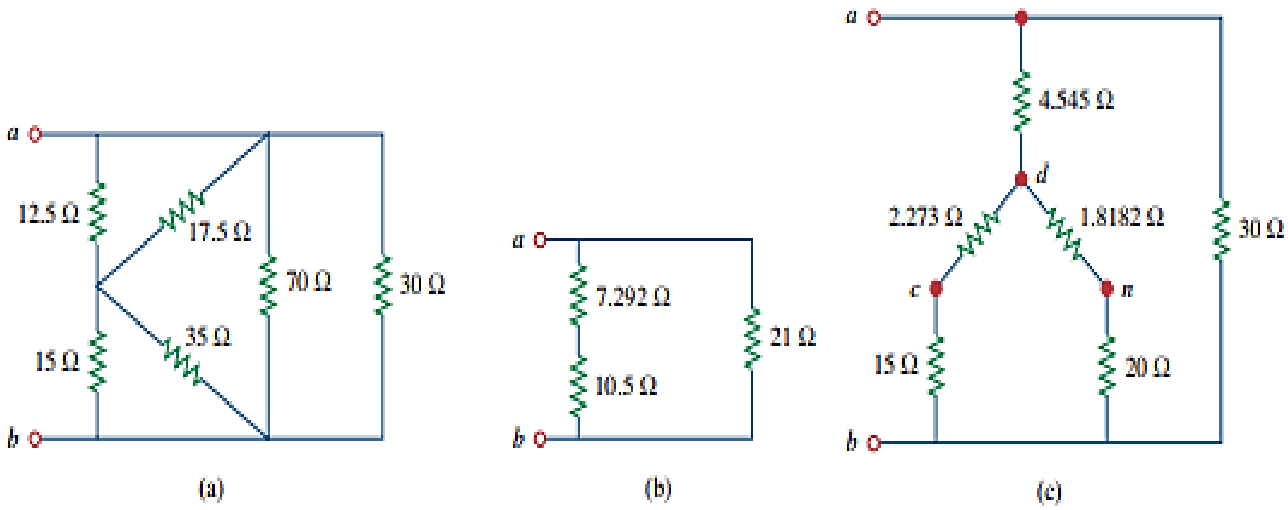
$$R_2 = \frac{R_cR_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$

$$R_3 = \frac{R_aR_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$

Example (2): find Rab use it to find I in the figure :



Solution:



$$70 \parallel 30 = \frac{70 \times 30}{70 + 30} = 21\ \Omega$$

$$12.5 \parallel 17.5 = \frac{12.5 \times 17.5}{12.5 + 17.5} = 7.292\ \Omega$$

$$15 \parallel 35 = \frac{15 \times 35}{15 + 35} = 10.5\ \Omega$$

$$R_{ab} = (7.292 + 10.5) \parallel 21 = \frac{17.792 \times 21}{17.792 + 21} = \mathbf{9.632\ \Omega}$$

Then

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = \mathbf{12.458\ A}$$

$$R_{ad} = \frac{R_c R_n}{R_a + R_c + R_n} = \frac{10 \times 12.5}{5 + 10 + 12.5} = 4.545\ \Omega$$

$$R_{cd} = \frac{R_a R_n}{27.5} = \frac{5 \times 12.5}{27.5} = 2.273\ \Omega$$

$$R_{nd} = \frac{R_a R_c}{27.5} = \frac{5 \times 10}{27.5} = 1.8182\ \Omega$$

$$R_{db} = \frac{(2.273 + 15)(1.8182 + 20)}{2.273 + 15 + 1.8182 + 20} = \frac{376.9}{39.09} = 9.642\ \Omega$$

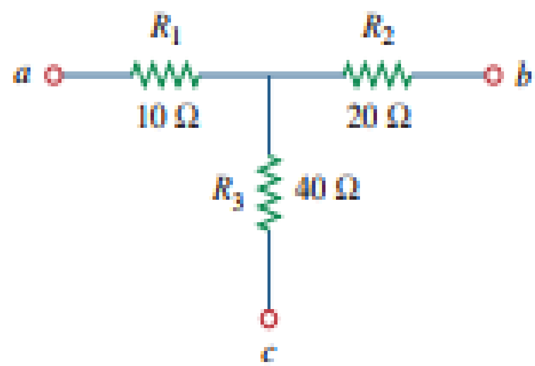
$$R_{ab} = \frac{(9.642 + 4.545)30}{9.642 + 4.545 + 30} = \frac{425.6}{44.19} = 9.631\ \Omega$$

This now leads to

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.631} = \mathbf{12.46\ A}$$

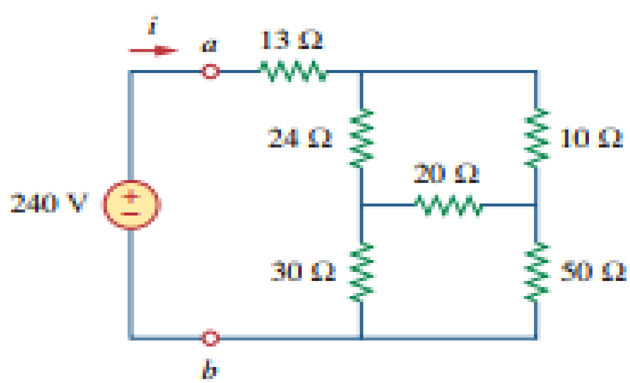
Problems

1. ***Transform the wye network in Figure below to a delta network:***



Answer: $R_a = 140\ \Omega, R_b = 70\ \Omega, R_c = 35\ \Omega.$

2. ***For the bridge network in Figure below , find R_{ab} and I.***



Answer: 40 Ω, 6 A.