Advance

Calculuse

the first course

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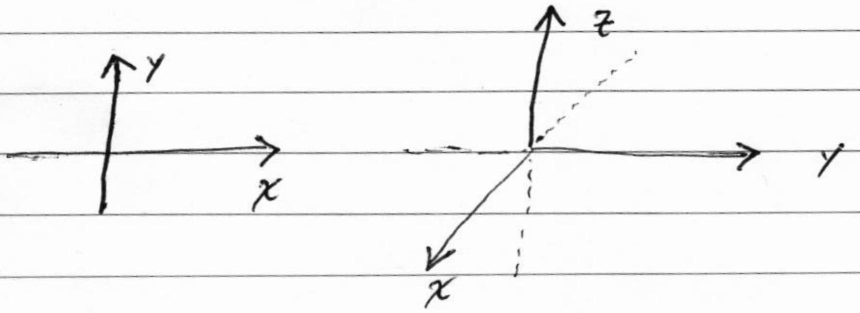
Vectors



The vector $\vec{V} = iV_1 + jV_2 = \langle V_1, V_2 \rangle$

$\vec{V} = iV_1 + jV_2 + kV_3 = \langle V_1, V_2, V_3 \rangle$

where V_1, V_2 are V components in R^2 space

and V_1, V_2, V_3 " " " " R^3 " .

~~where~~ The unit vector $\vec{u} = \langle 0, 1 \rangle$ or

$\vec{u} = \langle 1, 0 \rangle$ or

$\vec{u} = \langle \cos\theta, \sin\theta \rangle$

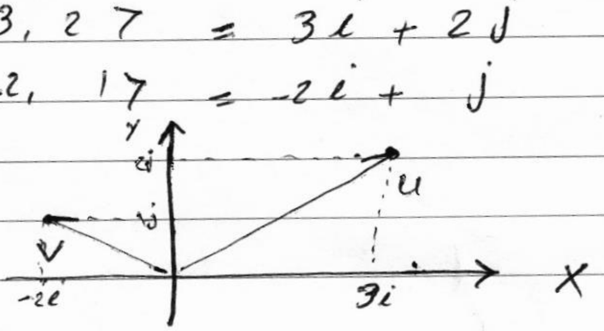
The length of vector $\|\vec{V}\|$ (norm of V)

$\|\vec{V}\| = \sqrt{V_1^2 + V_2^2}$ or $\|\vec{V}\| = \sqrt{V_1^2 + V_2^2 + V_3^2}$

ex

$\vec{V} = \langle 3, 2 \rangle = 3i + 2j$

$\vec{u} = \langle -2, 1 \rangle = -2i + j$



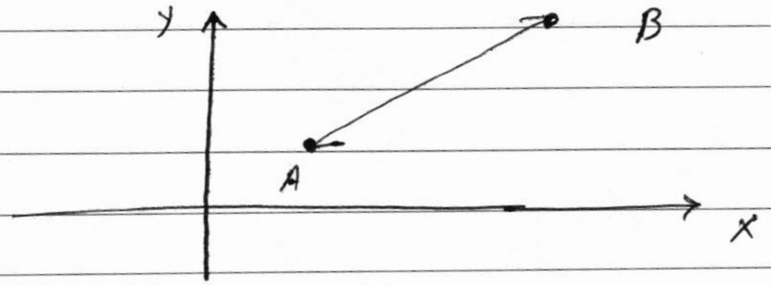
note

The vector $\vec{V}$ between any two point $A = (a_1, a_2)$

$B = (b_1, b_2)$

$$\vec{V} = \vec{AB} = \langle b_1 - a_1 , b_2 - a_2 \rangle$$

$A = (1,1)$
$B = (3,4)$



$$\vec{V} = \vec{AB} = \langle 3-1 , 4-1 \rangle = \langle 2, 3 \rangle$$

$$\vec{u} = \vec{BA} = \langle 1-3 , 1-4 \rangle = \langle -2, -3 \rangle$$

clear That

$$\vec{u} \neq \vec{V}$$

but

$$\| u \| = \| V \| \quad \text{since}$$

$$\| u \| = \sqrt{-2^2 + -3^2} = \sqrt{13}$$

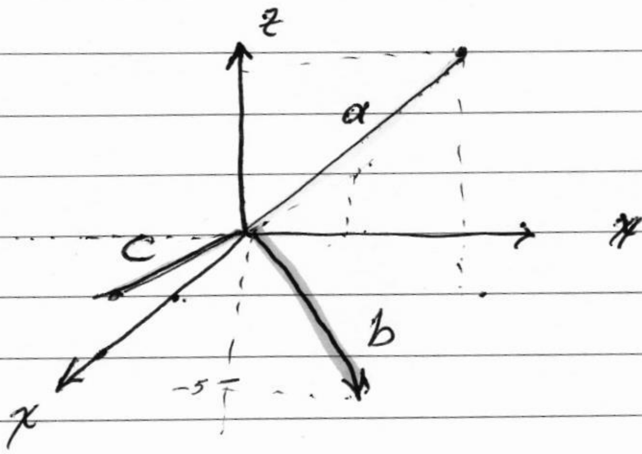
$$\| V \| = \sqrt{2^2 + 3^2} = \sqrt{13}$$

ex in ($\mathbb{R}^3$)

let $\vec{a} = \langle 2, 6, 7 \rangle \Longrightarrow \|a\| = \sqrt{89}$

$\vec{b} = \langle -2, 4, -5 \rangle \Longrightarrow \|b\| = \sqrt{45}$

$\vec{c} = \langle 4, 0, 1 \rangle \Longrightarrow \|c\| = \sqrt{17}$

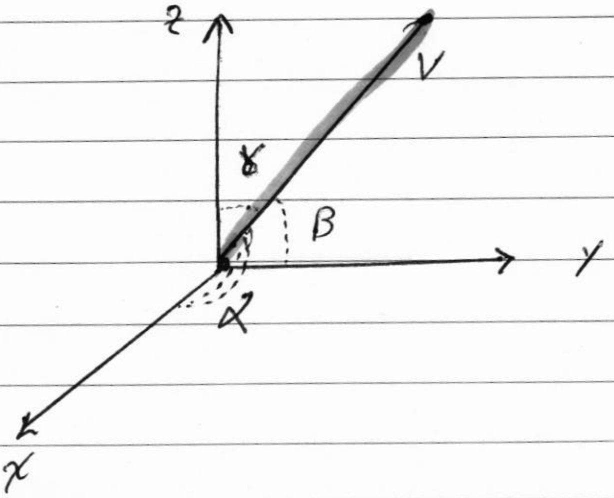


The angle in $\mathbb{R}^3$ is compute as follows :-

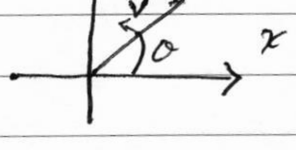
$$\cos \alpha = \frac{V_1}{\|V\|}$$

$$\cos \beta = \frac{V_2}{\|V\|}$$

$$\cos \gamma = \frac{V_3}{\|V\|}$$



While in $\mathbb{R}^2$



$0 < \theta < \pi$

ante cloc wise

Operations on vectors

① Addition

let $\bar{A} = \langle a_1, a_2 \rangle$ or $\langle a_1, a_2, a_3 \rangle$

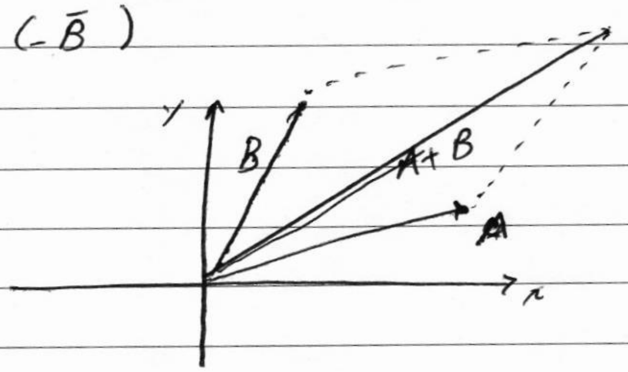
$\bar{B} = \langle b_1, b_2 \rangle$ or $\langle b_1, b_2, b_3 \rangle$

if They have The same intial point Then:-

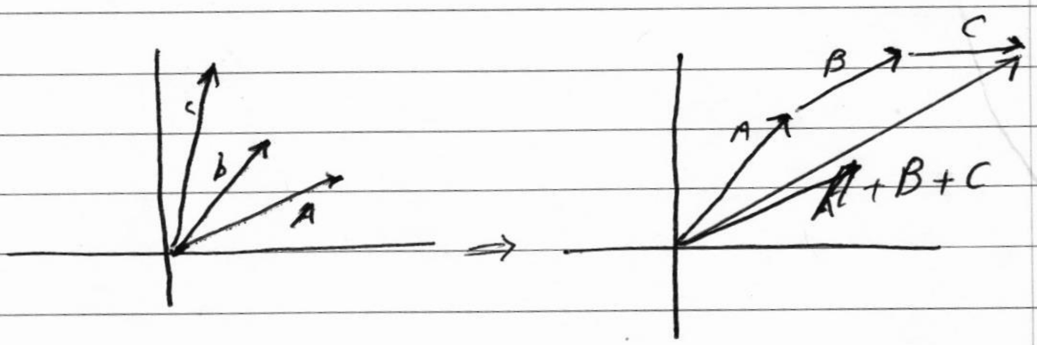
define $\bar{A} + \bar{B} = \langle a_1 + b_1, a_2 + b_2 \rangle$ or $\langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$

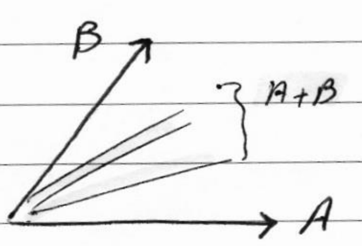
and $\bar{A} - \bar{B} = \bar{A} + (-\bar{B})$

Geomatrically

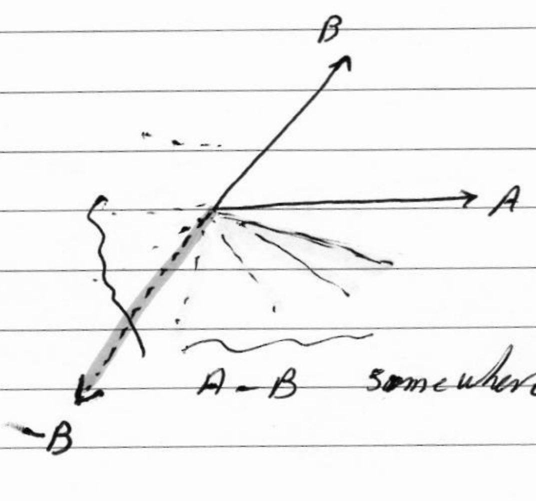


or





some where between Them



A - B somewhere between A and -B

2) Product

a) dot product "."

$\bar{A} = \langle a_1, a_2 \rangle$
$\bar{B} = \langle b_1, b_2 \rangle$
$\Rightarrow \bar{A}\cdot\bar{B} = a_1 \cdot b_1 + a_2 \cdot b_2$
$\bar{A}\cdot\bar{B} = \|A\| \cdot \|B\| \cos\theta$

ex

let $\bar{A} = \langle 2, -3 \rangle$, $\bar{B} = \langle 1, 2 \rangle$. Find (a) $\bar{A}\cdot\bar{B}$ (b) angle betwen Them

(a)

$$\bar{A}\cdot\bar{B} = (2\cdot 1) + (-3\cdot 2) = -4$$

(b)

$$-4 = \sqrt{13}\cdot\sqrt{5}\,\cos\theta$$

$$\Rightarrow \cos\theta = \frac{-4}{\sqrt{65}} \Longrightarrow \theta = \cos^{-1}\left(-\frac{4}{\sqrt{65}}\right)$$

ex2 prove That: $\bar{A} \perp \bar{B}$ if and only if $\bar{A}\cdot\bar{B} = 0$

sol:

let $\bar{A} \perp \bar{B} \Rightarrow \theta = 90^\circ \Rightarrow \bar{A}\cdot\bar{B} = \|A\|\cdot\|B\|\cdot\cos 90$
$= 0$

Now let $\bar{A}\cdot\bar{B} = 0$

$\Rightarrow \|A\|\cdot\|B\| \cos\theta = 0$

$\Rightarrow \cos\theta = 0 \Rightarrow \theta = \cos^{-1} 0 = 90^\circ \Rightarrow \bar{A} \perp \bar{B}$

(b) Croos product "$\times$"

$\bar{A} = \langle a_1, a_2, a_3 \rangle$

Then

$\bar{B} = \langle b_1, b_2, b_3 \rangle$

$$\bar{A} \times \bar{B} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

<u>ex</u> let $\bar{u} = \langle 1, 2, -2 \rangle$
$\bar{V} = \langle 3, 0, 1 \rangle$

$$\bar{u} \times \bar{V} = \begin{vmatrix} i & j & k \\ 1 & 2 & -2 \\ 3 & 0 & 1 \end{vmatrix}$$

$$= i \begin{vmatrix} 2 & -2 \\ 0 & 1 \end{vmatrix} - j \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} + k \begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix}$$

$$= 2i - 7j - 6k$$

<u>propertics of croos product</u>

(1) $\bar{A} \times \bar{B} \perp \bar{A}$ and $\bar{A} \times \bar{B} \perp \bar{B}$

(2) $\bar{A} \times \bar{B} = 0 \iff \bar{A} \parallel \bar{B}$ $\quad (\bar{A} \times \bar{A} = 0)$

(3) $\bar{A} \times \bar{B} = -(\bar{B} \times \bar{A})$ i.e $\bar{A} \times \bar{B} \neq \bar{B} \times \bar{A}$

(4) $\bar{u} \times (\bar{A} + \bar{B}) = \bar{u} \times \bar{A} + \bar{u} \times \bar{B}$

(5) $(\bar{A} \times \bar{B}) \cdot 5 = 5\bar{A} \times \bar{B} = \bar{A} \times 5\bar{B}$

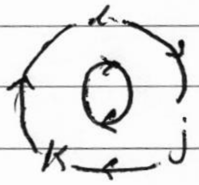
notes for croos products

(1)

$i \times j = K \quad , \quad j \times i = -K$

$j \times K = i \quad , \quad K \times j = -i$

$K \times i = j \quad , \quad i \times K = -j$



(2) $\|\bar{A} \times \bar{B}\|$ = Area of ~~Trapezium~~ equiletral Generated by $\bar{A}$ and $\bar{B}$.

(3) $\|\bar{A} \times \bar{B}\|^2 = \|A\|^2 \|B\|^2 - (\bar{A} \cdot \bar{B})^2$

~~AxB = Area of Trapezium between A and B~~

Lagrangs identity

properties of dot product

(1) $i \cdot i = j \cdot j = K \cdot K = 1$ & $i \cdot j = i \cdot K = j \cdot K = 0$

(2) $\bar{u} \cdot \bar{v} = \bar{v} \cdot \bar{u}$

(3) $\bar{u} \cdot (\bar{A} + \bar{B}) = \bar{u} \cdot \bar{A} + \bar{u} \cdot \bar{B}$

(4) $s \cdot (\bar{A} \cdot \bar{B}) = s\bar{A} \cdot \bar{B} = \bar{A} \cdot s\bar{B}$

(5) $\bar{A} \cdot \bar{A} = \|A\|^2$

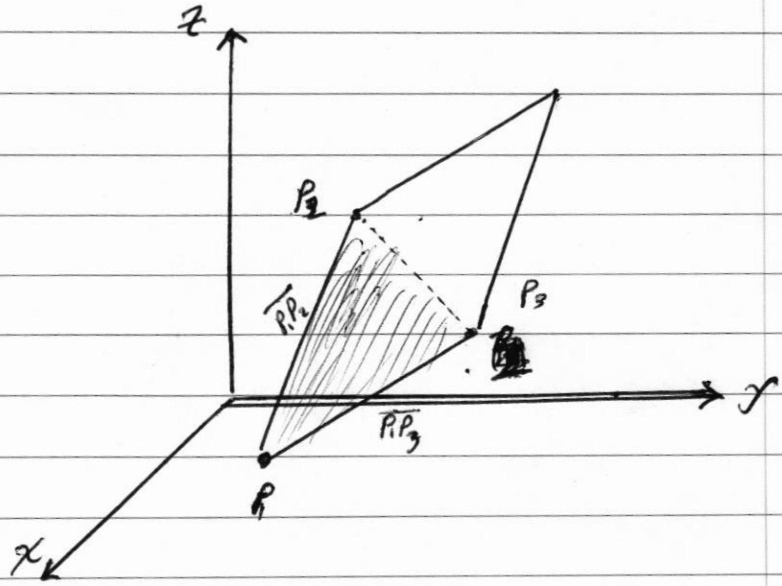
ex 8

Find The area of Triangle its heads The points

$P_1 = (2, 2, 0)$

$P_2 = (-1, 0, 2)$

$P_3 = (0, 4, 3)$



$\overline{P_1P_2} = \langle -3, -2, 2 \rangle$

$\overline{P_1P_3} = \langle -2, 2, 3 \rangle$

$$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix} i & j & k \\ -3 & -2 & 2 \\ -2 & 2 & 3 \end{vmatrix} = -10i + 5j - 10k$$

$$\| \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} \| = \sqrt{100 + 25 + 100} = 15$$

area of $P_1P_2P_3 \, \Delta = \frac{1}{2} \cdot 15 = 7.5$

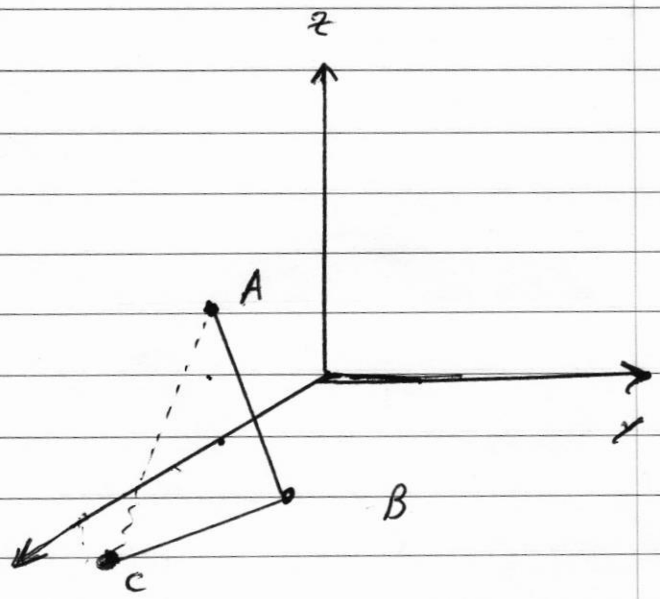
H.W.

prove That $\bar{A} \times \bar{B} = 0 \overset{iff}{\Longleftrightarrow} \bar{A} \parallel \bar{B}$

ex

prove That : The points $\begin{cases} A = (2, -1, 1) \\ B = (3, 2, -1) \\ C = (7, 0, -2) \end{cases}$ construct a right Triangle from right side.

sol.



$\overline{AB} = \langle 1, 3, -2 \rangle$

$\overline{BC} = \langle 4, -2, -1 \rangle$

$\overline{AB} \cdot \overline{BC} = 4 - 6 + 2 = 0$

$\therefore \overline{AB} \perp \overline{BC}$

or one can use pythagors Theorem

$\overline{AC} = \langle 5, 1, -3 \rangle$

$\Longrightarrow \|\overline{AC}\|^2 = 35 \quad \& \quad \|\overline{AB}\|^2 = 14 \quad \& \quad \|\overline{BC}\|^2 = 21$

$\therefore \|\overline{AC}\|^2 = \|\overline{AB}\|^2 + \|\overline{BC}\|^2$

$\therefore AB \perp BC$ $\square$