Triple scalar product (T. S. p.)

$$\bar{A} \cdot (\bar{B} \times \bar{C}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

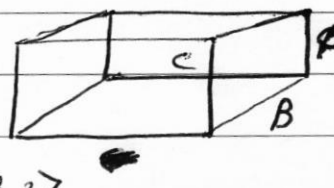
clear That

$$\bar{A} \cdot (\bar{B} \times \bar{C}) = \bar{B} \cdot (\bar{C} \times \bar{A}) = \bar{C} \cdot (\bar{A} \times \bar{B})$$

T.S.P evalute The volume of cubic $\bar{A}, \bar{B}, \bar{C}$
parallelogram

ex

let The parallelogram its sides



$\bar{A} = \langle 3, 2, 1 \rangle$, $\bar{B} = \langle 1, 1, 2 \rangle$, $\bar{C} = \langle 1, 3, 2 \rangle$

comput: (a) Volum, (b) area of Triangle between A & c
(c) find The angle between $\bar{A}$ and The plan CB

sol.

(a) $\bar{A} \cdot (\bar{B} \times \bar{C}) = \begin{vmatrix} 3 & 2 & 1 \\ 1 & 1 & 2 \\ 1 & 3 & 2 \end{vmatrix} = -10 \Rightarrow V = 10$ lower line

(b) $\bar{A} \times \bar{C} = \begin{vmatrix} i & j & k \\ 3 & 2 & 1 \\ 1 & 3 & 2 \end{vmatrix} = i - 5j + 7k \Rightarrow \| \bar{A} \times \bar{C} \| = \sqrt{75}$

$\Rightarrow$ area of $\Delta = \sqrt{75}/2$

(c) $\underset{\text{plan}}{\bar{C} \times \bar{B}} = \begin{vmatrix} i & j & k \\ 1 & 3 & 2 \\ 1 & 1 & 2 \end{vmatrix} = 4i - 2k \Rightarrow \| \bar{C} \times \bar{B} \| = \sqrt{20}$

$$A \cdot (\bar{C} \times \bar{B}) = \|A\| \cdot \|\bar{C} \times \bar{B}\| \cos\theta$$

$$10 = \sqrt{14} \cdot \sqrt{20} \cos\theta$$

$$\sqrt{\frac{5}{14}} = \cos\theta \Longrightarrow \theta = \cos^{-1} \sqrt{\frac{5}{14}}$$

$\square$

2016 - 10 - 16

Parametric equation

for Lines in R^2 and R^3

<u>Def.</u>

The line "L" in R^2 (R^3) That passes through the point $P_0 = (x_0, y_0)$ $(P_0 = (x_0, y_0, z_0))$ and parallel to the non-zero vector $\vec{V} = \langle a, b \rangle$ $(\vec{V} = \langle a, b, c \rangle)$

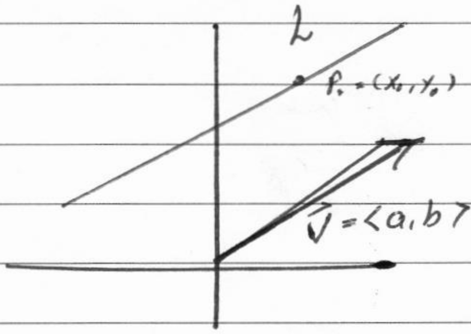
has The following parametric equation :-

L:
$$x = x_0 + at$$
$$y = y_0 + bt$$

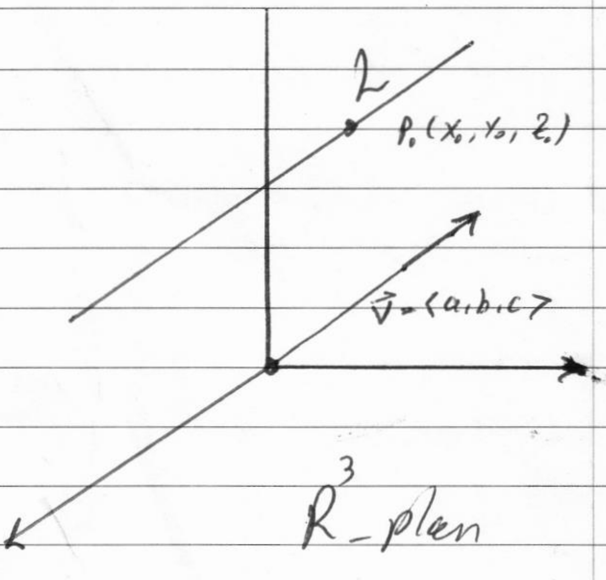
or in R^3

L:
$$x = x_0 + at$$
$$y = y_0 + bt$$
$$z = z_0 + ct$$

$t \in (-\infty, \infty)$



R^2 _ plan



R^3 _ plan

ex1 find The parametric eq. of The lines:-

1) passing through (4,2) and parallel to $\vec{v} = \langle -1, 5 \rangle$

2) " " (1,2,-3) " " " $\vec{v} = \langle 4, 5, -7 \rangle$

3) " " (0,0,0) " " " $\vec{v} = \langle 1, 1, 1 \rangle$

sol.

$L:$ 1) $x = 4 - t$, $y = 2 + 5t$

$L:$ 2) $x = 1 + 4t$, $y = 2 + 5t$, $z = -3 - 7t$

$L:$ 3) $x = t$, $y = t$, $z = t$

ex2 a) find The parametric eq. of The line L which passing through the points $p = (2, 4, -1)$ and $q = (5, 0, 7)$

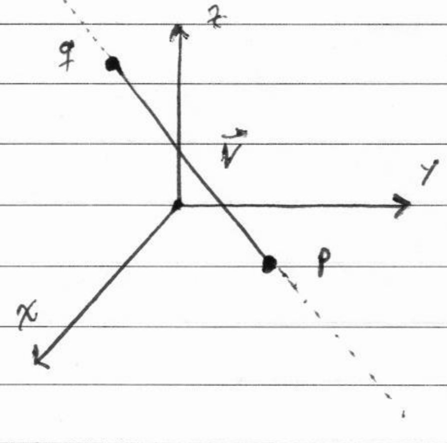
b) When dose The line intersect The xy-plan (R^2)

c) the parametric eq. of the line segment from p to q.

sol.

a) $\vec{v} = \overrightarrow{pq} = \langle 5-2, 0-4, 7+1 \rangle = \langle 3, -4, 8 \rangle$

$$\Rightarrow L: \begin{cases} x = 2 + 3t \\ y = 4 - 4t \\ z = -1 + 8t \end{cases}$$



b) intersect R^2 in $z = 0$

$\Rightarrow$

$0 = -1 + 8t \Rightarrow t = \frac{1}{8}$

$$\Rightarrow \left. \begin{array}{l} x = 2 + \frac{3}{8} \\ y = 4 - \frac{4}{8} \end{array} \right\} \Rightarrow \text{intersect at } p = \left(\frac{19}{8}, \frac{28}{8} \right)$$

c)

The Line $L_1 = \begin{cases} x = 2 + 3t \\ y = 4 - 4t \\ z = -1 + 8t \end{cases}$ pass through p

at $t = 0$ and through q at $t = 1$

by use any point coordinates x or y or z.

so; The eq. becom for The line segement $\overline{pq}$

$$L_2: \begin{cases} x = 2 + 3t \\ y = 4 - 4t \\ z = -1 + 8t \end{cases} \qquad 0 \leq t \leq 1$$

ملاحظة

① لايجاد معادلة مستقيم يجب ان يكون لدينا نقطة على المستقيم ومتجه ~~[illegible]~~ . يوازي المستقيم .

② في حالة وجود نقطتين فيمكن ايجاد المتجه الذي يحويهما ثم ناخذ احدهما مع المتجه لايجاد المعادلة . واذا طلب تحديد المعلمة t لقطعة المستقيم نحددها باستخدام احد مساقط النقاط كما فعلنا في المثال السابق .

ex 3

Let L_1 & L_2 be two lines with

$$L_1: \begin{cases} x = 1 + 4t \\ y = 5 - 4t \\ z = -1 + 5t \end{cases} \qquad L_2: \begin{cases} x = 2 + 8t \\ y = 4 - 3t \\ z = 5 + t \end{cases}$$

a) are The lines parallel ?
b) dose " " intersect ?

Sol.

a) $\vec{V}_1 = \langle 4, -4, 5 \rangle$
$\vec{V}_2 = \langle 8, -3, 1 \rangle$

$$\Rightarrow \quad \vec{V}_1 \times \vec{V}_2 = \begin{vmatrix} i & j & k \\ 4 & -4 & 5 \\ 8 & -3 & 1 \end{vmatrix} = \langle 11, 36, 20 \rangle \neq 0$$

$$\Rightarrow \vec{V}_1 \not\parallel \vec{V}_2 \Rightarrow L_1 \not\parallel L_2$$

b) if the lines intersect Then There is a point $P(x, y, z)$ satisfies both of L_1 & L_2 so that :-

$x_1 = x_2, \; y_1 = y_2, \; z_1 = z_2$

$$\left.\begin{aligned} 1 + 4t_1 &= 2 + 8t_2 \quad (1) \\ 5 - 4t_1 &= 4 - 3t_2 \quad (2) \\ -1 + 5t_1 &= 5 + t_2 \quad (3) \end{aligned}\right\} \xrightarrow{1+2} 6 = 6 + 5t_2 \Rightarrow t_2 = 0 \quad (4)$$

$$(4) \text{ in } (1) \Rightarrow t_1 = \frac{1}{4} \quad (5) \quad \xrightarrow{(5) \text{ in } (3)} -1 + \frac{5}{4} = 5 \Rightarrow 5 = 24 \Rightarrow C!$$

$\therefore L_1$ not intersect L_2 $\square$

H.W. تحديد علاقة الخطين

def

The distance between line L_1: $x = x_0 + at$ —— ①

$y = y_0 + bt$

$z = z_0 + ct$

and The point $p = (\alpha, \beta, \gamma)$ can be computed

as follows :-

① find The eq. $D = \Sigma ($

$$D(t) = (x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2 \quad \text{—— ②}$$

② find t by set $\frac{dD}{dt} = 0$

③ find coordinate of L_1 using t (say (Q))

④ comput $\sqrt{D}$ again with out "t" $\equiv d(P, Q)$

ex find The distance between $p = (1,1,5)$ and L_1

$x = 1+t$, $y = 3-t$, $z = 2t$

sol.

step① $D(t) = (1+t-1)^2 + (3-t-1)^2 + (2t-5)^2$

$= 6t^2 - 24t + 29$

step ② $\frac{dD}{dt} = 0 = 12t - 24 \implies t = 2$

step3 $L_1 = (1+2, 3-2, 2 \cdot 2) = (3, 1, 4)$

step4

$$D = (3-1)^2 + (1-1)^2 + (4-5)^2 = 4 + 0 + 1$$

$$\sqrt{D} = \sqrt{5}$$

method (2) to find the distance between The points and The Line L (passing throug P and parallel to vector V ~~and the points~~.

$$d = \frac{\| \vec{PS} \times V \|}{\| V \|}$$

ex recall The above example $P = (1,1,5)$

$$L : \begin{cases} x = 1 + t \\ y = 3 - t \\ z = 2t \end{cases}$$

sol.

from L $S = (1,3,0)$, $V = \langle 1, -1, 2 \rangle$

$\Rightarrow \vec{PS} = (0,-2,5)$

$\Rightarrow \vec{PS} \times V = \begin{vmatrix} i & j & k \\ 0 & -2 & 5 \\ 1 & -1 & 2 \end{vmatrix} = \langle 1, 5, 2 \rangle$

$\Rightarrow \| \vec{PS} \times V \| = \sqrt{1+25+4} = \sqrt{30}$

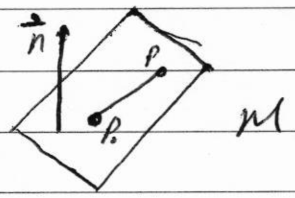
$\&$ $\| V \| = \sqrt{1+1+4} = \sqrt{6}$

$\Rightarrow$

$$d = \frac{\sqrt{30}}{\sqrt{6}} = \sqrt{\frac{30}{6}} = \sqrt{5}$$

Equation of plane in space

A plane M in space can be determined by knowing a point on plan (P_0) and a vector $\vec{n}$ that perpendicual perpendicular to M, since $\forall P \in M$



$\overrightarrow{P_0P} \cdot \vec{n} = 0$ —— (1) where $\begin{cases} \vec{n} = \langle A, B, C \rangle \\ P = (x, y, z) \end{cases}$

$\Rightarrow$

$(Ai + Bj + Ck) \cdot (x - x_0\, i + y - y_0\, j + z - z_0\, k) = 0$

$\Rightarrow A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$ —— (2)

$\Rightarrow Ax + By + Cz = d$ —— (3)

(1) is called vector equtions.

(2) " " component equtions.

(3) " " simplified " " .

ملاحظة

لإيجاد معادلة المستوى نحتاج الى نقطة في المستوى ومتجه عمودي
على المستوى ويمكن ايجاد المتجه العمودي اذا علمت ثلاث نقاط ليست على
استقامة واحدة وكذلك اذا علم متجهين ونقطة .

ex1 find plane eq. That perpendicular To vector

$\vec{n} = \langle 5, 2, -1 \rangle$ and pass Through point $(-3, 0, 7)$

sol.

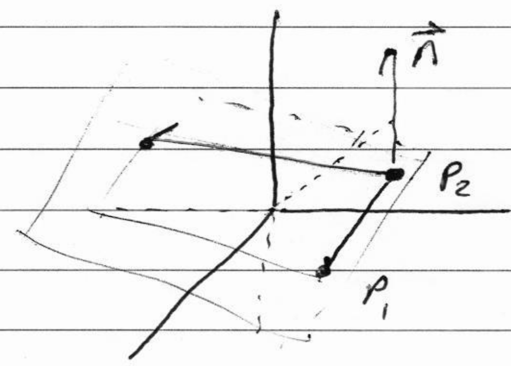
$$5(x+3) + 2(y-0) - 1(z-7) = 0$$

$$5x + 2y - z = -15 - 7 = -22$$

3 ليس على استقامة واحده

ex2 find an eq. for The plane Through Tr colinear

$P_1(1, 2, -1)$, $P_2(2, 3, 1)$, $P_3(3, -1, 2)$



$\overrightarrow{P_1P_2} = \langle 1, 1, 2 \rangle$

$\overrightarrow{P_2P_3} = \langle 1, -4, 1 \rangle$

$$\vec{n} = \overline{P_1P_2} \times \overline{P_2P_3} = \begin{vmatrix} i & j & k \\ 1 & 1 & 2 \\ 1 & -4 & 1 \end{vmatrix}$$

$$= \langle 9, 1, -5 \rangle$$

لا يهم اي نقطه

now use one of the point (it does not matter which)

$$9(x-2) + (y-3) - 5(z-1) = 0$$

$$9x + y - 5z = 16$$

H.w. same ex2 but $P_1(0, 0, 1)$, $P_2(2, 0, 0)$, $P_3(0, 3, 0)$

ex

Is The Line $L_1 : \begin{cases} x = 3 + 8t \\ y = 4 + 5t \\ z = -3 - t \end{cases}$ parallel to plane M

$x - 3y + 5z = 12$? if not find points of intersection.

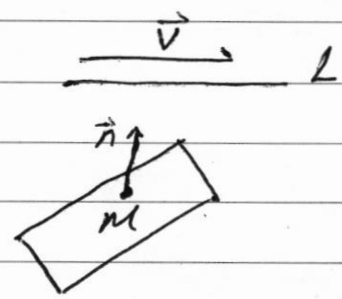
sol.

$\vec{V}$ parallel to L_1 is $\vec{V} = \langle 8, 5, -1 \rangle$

$\vec{n}$ perpendicular to M is $\vec{n} = \langle 1, -3, 5 \rangle$

$\vec{V} \cdot \vec{n} = 8 - 15 - 5 = -12 \neq 0$

$\therefore$ m not parallel to L_1



Let $P_0 (x_0, y_0, z_0)$ is The point of inter.

$\Longrightarrow$

$x_0 - 3y_0 + 5z_0 = 12$ ——— ①

$x_0 = 3 + 8t \quad , \quad y_0 = 4 + 5t \quad , \quad z_0 = -3 - t$ ——— ②

② in ① $\Rightarrow (3+8t) - (4+5t) + 5(-3-t) = 12$

$\Rightarrow -12t - 24 = 12 \Rightarrow t = -3$ ——— ③

③ in ② $\Rightarrow \left. \begin{array}{l} \cancel{x}_0 = (-21) \\ y_0 = (-11) \\ z_0 = 0 \end{array} \right\} \Rightarrow P_0 (-21, -11, 0)$